A Practical Compress-and-Forward Relay Scheme Based on Superposition Coding

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Abstract—**Compress and forward(CF) is one of the protocols in cooperative communication and has drawn much attention. In CF scheme, the relay compresses and transmits the received signals to the destination, where Wyner-Ziv coding is considered as an efficient way to compress the signal with side information available. Since the current research of superposition coding for Wyner-Ziv problem only remains in theory, we design a practical CF scheme in half-duplex mode by using the technique of superposition coding and provide specific coding strategy for it. The relay compresses the received signal by the quantization method based on superposition coding, and then the destination performs a joint decoding to recover the original message. Simulations show that our proposed scheme outperforms the conventional CF scheme with scalar quantization at the relay. This coding strategy can be easily extended to the scenarios of fading channels, which are more practical in wireless communication.**

Keywords - compress-and-forward; relay channel; superposition coding; quantization; Wyner-Ziv coding

I. INTRODUCTION

In the last several years, there has been an upsurge of interest in cooperative communication. Using multiple nodes in a wireless network can obtain an increase in diversity and power saving as compared to using only the direct link from source to destination. Among the proposed schemes in cooperative communication, the three-terminal relay channel is a fundamental model [1]. Although the capacity of the general relay channel is still not known, the achievable rates of several relaying protocols were proposed in [2]. According to what the relay should do after receiving the signals, these protocols are classified into decode-and-forward (DF), amplify-and-forward (AF), and compress-and-forward (CF).

Although DF is efficient in some scenarios, the achievable rate is determined by the capacity of the channel between the source and relay, because the relay must perfectly decode the source message. To alleviate this problem, CF has been proposed, where the relay does not attempt to decode the signal of the source. Instead, it implements Wyner-Ziv coding [3] (or source coding with side information at the decoder) to compress the signals. The signals that received at the relay and the destination are correlated because they are different noisy versions of the same source signal. Thus, the relay can exploit this correlation to compress the received signal at the relay without knowing the received signal at the destination. At the destination, the signal is recovered through joint decoding of the received signals from both the relay and the destination. CF outperforms DF when the signals that received at the relay and the destination are highly correlated, e.g., when the relay is close to the destination. In addition, as opposed to DF, CF is always more desirable than direct transmission. Therefore, the relay can always aid the source in CF scheme, even if the link between the source and the relay is poor.

Generally speaking, researches in the area of practical CF relaying can be divided into two categories. Works in the first category attempt to design a reliable CF scheme to approach the capacity bound and implement Wyner-Ziv coding faithfully at the relay [4–7]. The second category, which is closer to our research, consists of works that focus on the design of relay quantization [8, 9]. In this paper, we investigate a practical CF scheme based on quantization by superposition coding. In [10], the authors analyzed the performance of superposition coding for Wyner-Ziv problem and then particularized for the binary and the Gaussian case. We extend this analysis to CF scheme and design a practical coding strategy.

The remainder of this paper is organized as follows. Section 2 presents the CF system model in half-duplex mode. In section 3, we analyze the framework of the relay quantization based on superposition coding and propose a practical coding strategy. Some simulation results about our proposed CF scheme are shown in Section 4, and Section 5 concludes the paper.

II. SYSTEM MODEL

The wireless relay channel, as shown in Fig. 1, consists of one source (S), one relay (R) and one destination (D). The coefficient h_{SR} denotes the channel gain between the source and the relay. h_{RD} , h_{SD} are defined similarly. The source sends data to the destination with the aid of the relay that does not have its own data to transmit. We assume that the relay works in half-duplex mode, which means that it cannot transmit and receive simultaneously in the same frequency band or in the same time slot. CF scheme in full-duplex mode is difficult to design because it requires isolation or accurate interference cancellation between transmitted and received signals that differ in power by multiple orders of magnitude. Instead, half-duplex is a simpler and more realistic approach. Thus, in this paper, we will focus the scheme in half-duplex mode. Without

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loss of generality, we assume that half-duplexing is performed in time-division, so that the process of transmission takes place over two slots of time fractions t and $t' = 1-t$. In the first time slot, S transmits a signal, which is corrupted and received by R and D. We call this the broadcast (BC) mode of communication. In the second time slot, both S and R transmit signals to D. This pattern can be called the multiple access (MAC) mode. These two modes are showed in Fig. 2. The variables X, V, W, Y denote the source transmitted signal, the relay received signal, the relay transmitted signal, and the destination received signal respectively. The subscripts 1 and 2 mean BC mode and MAC mode respectively.

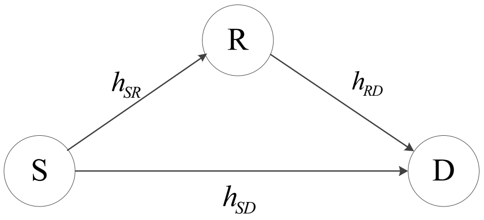

Figure 1. The relay channel with three nodes

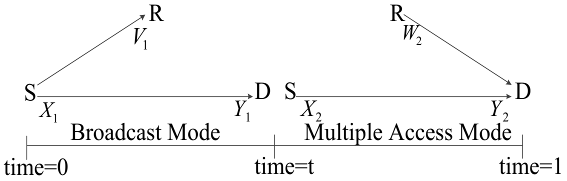

Figure 2. Broadcast (BC) and multiple access (MAC) modes of the half-duplex relay channel.

The Gaussian half-duplex relay channel model can be illustrated by the following expressions:

BC mode:

$$Y_1 = h_{SD} X_1 + N_{D1}, \tag{1}$$

$$V_1 = h_{SR} X_1 + N_{R1}, \tag{2}$$

MAC mode:

$$Y_2 = h_{SD} X_2 + h_{RD} W_2 + N_{D2}. \tag{3}$$

In the above model, N_{D1} denotes the noise at the destination in BC mode. N_{R1}, N_{D2} can be similarly interpreted. Other expressions are mentioned previously.

Then we present the power constraint in the half-duplex relay channel. In (1)-(3), we define the power for the signals as follows:

$$E(X_1^2) = P_{S_1}, \tag{4}$$

$$E(X_2^2) = P_{S_2}, \tag{5}$$

$$E(W_2^2) = P_{R_2}. \tag{6}$$

And these signal powers are constrained by the following expression:

$$tP_{S_1} + (1-t)(P_{S_2} + P_{R_2}) \le P_{sum} \tag{7}$$

where P_{sum} is the total signal power of the system. The variables $P_{S_1}, P_{S_2}, P_{R_2}$ represent the power of the source transmitted signal during the first time slot, the power of the source transmitted signal during the second time slot, and the power of the relay transmitted signal during the second time slot respectively.

In half-duplex CF scheme, the length of the symbols in each time slot is constrained as follows:

$$qN_1 / R_2 = N_2, \tag{8}$$

$$N_1 = tN, \tag{9}$$

$$N_2 = (1-t)N. \tag{10}$$

The parameter q denotes the quantization bits at the relay, N_1 and N_2 mean the length of symbols in each two time slots respectively, N means the total length of symbols, R_2 represents the rate in relay-destination link.

III. THE DESIGN OF THE CF SCHEME BASED ON SUPERPOSITION CODING

In this section, we present the details of the designed CF scheme, based on the half-duplex operation explained in Section 2. First, we mention several simplifications to the model. Then the analysis of the quantizer based on superposition coding is presented. Finally, we describe the specific coding strategy of the scheme.

A. Simplifications for the half-duplex CF model

In this subsection, we present a few simplifications to the CF coding scheme described earlier. These simplifications greatly reduce the complexity of practical implementation.

We first assume the three nodes are in a straight line, which is usually assumed in current literature [9]. The system model is depicted in Fig. 3. The distance between S and D is normalized to unity, and R is assumed to lie in the straight line between S and D. We use d to represent the distance from the relay node to the source node. The colinearity of these three nodes does not affect the derivation of any of the results in this paper, but it enables a simple characterization of the relay position. In the above setting, the SD channel gain is $h_{SD} = 1$, the SR channel gain is $h_{SR} = 1/d^{\lambda}$, and the SR channel gain is $h_{SR} = 1/(1-d)^{\lambda}$, where λ denotes the channel attenuation exponent.

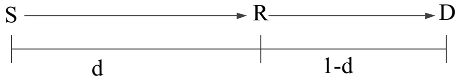

Figure 3. Relay channel with source, relay and destination in a straight line.

Second, in order to determine the power allocation and the time-division factor t in a specific case, we use the method of numerical analysis presented in [11]. In a wide range of relay positions and signal-to noise ratios (SNRs), the numerical results show that the achievable rate is maximized when all of the source signal power is allocated to BC mode. Therefore, in MAC mode, all of the power is allocated to the relay and the source remains silent. This result greatly simplifies the scheme design and eliminates the need for source-relay symbol synchronization, which is difficult to implement in practice.

Finally, as shown in [12], the relaying gain is substantial only at low SNRs. Therefore, over source-relay and source-destination links, we use binary modulation, which is known to be efficient at low SNRs and is used almost universally for low data-rate communications. This enables the use of capacity-approaching binary low density parity-check (LDPC) codes over these links. Unlike the source-relay and source-destination links, higher order modulation is necessary to achieve the capacity of the strong relay-destination link.

B. *Relay quantization based on superposition coding*

In this subsection, we present a practical analysis for the CF scheme using the technique of superposition coding. The works in [10] provide a foundation for our design.

From (1) and (2), the relationship between the two correlated signals at the relay and the destination can be derived as

$$Y_1 = \alpha V_1 + W \tag{11}$$

where $\alpha = h_{SD} / h_{SR}$ and $W = N_{D_1} - \alpha N_{R_1}$.

After the relay receiving signals from the source, unlike the other schemes, we use the method of superposition coding to accomplish the quantization process, based on the analysis of superposition coding for Wyner-Ziv problem in [10]. The quantization can be expressed as

$$C_1 + C_0 = \beta V_1 + U + Z \ . \tag{12}$$

The superposition codes are C_1 and C_0 . U is a dither signal known to both the relay and destination and is independent from other signals. Z is the quantization error of superposition coding. β is a coefficient that will be explained later. The relay sends code C_1 to the destination.

At the destination, the decoder evaluates

$$\begin{aligned}[][\beta Y_1 / \alpha + U - C_1]_{C_0} &= [C_0 - (Z - \beta W / \alpha)]_{C_0} \\ &= Z - \beta W / \alpha \end{aligned} \tag{13}$$

and finally reconstructs

$$\begin{aligned} \hat{V}_1 &= Y_1 / \alpha + \gamma [\beta Y_1 / \alpha + U - C_1]_{C_0} \\ &= V_1 + \gamma Z + (1 - \gamma\beta) W / \alpha \end{aligned} . \tag{14}$$

In order to minimize the power of noise between V_1 and $\hat{V}_1$, we set

$$\beta = \alpha \sqrt{1 - P_Z / P_W} \tag{15}$$

$$\gamma = (1 - \alpha^2 P_Z / P_W) / \beta \ , \tag{16}$$

then the power of $Z - \beta W / \alpha$ in (13) equals P_W , and the power of the noise between V_1 and $\hat{V}_1$ is minimized and approximately equals P_Z .

Proper choice of the two codes in superposition coding is necessary. In particular, the code C_1 must offer a good covering of the space, in other words, it must be good for source coding purposes. On the other hand, C_0 must take the role of a good channel code in order to avoid the decoding errors.

C. *The description of the coding strategy for the whole CF system*

In this section, we describe the coding strategy for the proposed CF scheme in detail. The schematic illustration of the scheme is depicted in Fig. 4.

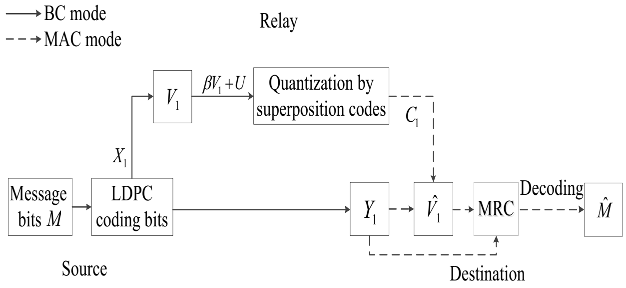

Figure 4. Schematic illustration of the proposed CF scheme.

- Source: The source is active only in the BC mode. It encodes the information bits with an LDPC channel code of rate R_1 . The resulting coding bits are mapped to binary phase-shift keying (BPSK) transmit symbols, which are broadcasted to the relay and the destination.
- Relay: In BC mode, the relay receives signal from the source, it then quantizes the received signal using the method of superposition coding described in Section 3.2. In MAC mode, the relay sends code C_1 to the destination.
- Destination: In BC mode, the destination receives signals from the source, as (1) shows. In MAC mode, after the destination receiving the signals from the relay, it first recovers the signal V_1 , using (14), and then it performs a maximum ratio combination (MRC) for the signals $\hat{V}_1$ and Y_1 to get the log-likelihood ratio (LLR). Finally, it uses belief propagation (BP) algorithm to decode the source message.

IV. SIMULATION RESULTS AND ANALYSIS

To illustrate the performance of the proposed CF scheme, we perform Monte Carlo simulations on additive white Gaussian noise (AWGN) channel and block fading channel. In these simulations, the three nodes are on a straight line as Section 3.1 notes. The distance from the source to the relay is set to $d = 0.75$. Given the total power of the system, we can obtain an optimal option of time-division factor and power allocation. We set the total power to -6dB, and then we obtain the optimal selection for these parameters: $t = 0.48$, $P_{S_1} = 1.777 P_{sum}$, $P_{R_2} = 0.282 P_{sum}$. The noise in (1)-(3) are all normalized zero mean Gaussians. Since noise power is normalized to unity, P_{sum} is also the equivalent relay channel SNRs in our plots. The channel attenuation exponent is chosen $\lambda = 1.5$. We choose the quantization bits $q = 2$. From (8)-(10), we can get $R_2 = 1.846$, so for the relay-destination link, quadrature phase-shift keying (QPSK) modulation is required.

A. AWGN channel

For the case of AWGN channel, the source encodes the message bits using a regular (3, 6) LDPC code of rate 1/2 and block length $N_1 = 10000$. For the superposition codes, we choose the code C_1 to be a trellis code for trellis coded quantization (TCQ) [13]. The number of states of the trellis is set to 8. Since code C_0 must be a good channel code in order to avoid the decoding errors in the operation in (13), we choose the code C_0 as a low rate convolutional code. It has rate 1/8 and constraint length $K = 11$ (i.e. $2^{10} = 1024$ states) as provided in [14]. The code's generator polynomials are given by the octal digits (2565, 2747, 3311, 3723, 2373, 2675, 3271, 2473). This code is mapped to $\pm\sqrt{P_W}$ for the quantization in real number field.

For comparison, we also perform simulations for CF scheme with scalar quantization, DF scheme, and direct link. For CF scheme with scalar quantization, the relay directly quantizes the received signal with quantization bits $q = 2$. For DF scheme, the relay decodes the received signals, re-encodes them and sends them to the destination. For the direct transmission, we allocate the total power to the source message in BC mode, for a fair comparison. Fig. 5 shows the bit error rate (BER) versus the total power of the system. For each point either 100 frame errors or 108 bits are simulated. From the results, at the BER of 10^{-4}, we can see that the proposed CF scheme based on superposition coding achieves approximately 0.4 dB over the CF scheme with scalar quantization and 0.8 dB over the DF scheme.

B. Block fading channel

Our analysis in Section 3 is based on the real AWGN channel. The same code designs are also useful in fading channels with perfect global knowledge of channel states at the encoder and decoder. Here we perform simulations for block fading channel, where the fading coefficient of each symbol remains constant during each block of transmission. At the source, we use a regular (3, 6) LDPC code of rate 1/2 and block length $N_1 = 1000$. Other parameters are the same as the case of AWGN channel. Frame error rate (FER) versus the total power of the system is depicted in Fig. 6, which shows that our proposed scheme performs better than the other schemes. For each point either 100 frame errors or 108 bits are simulated.

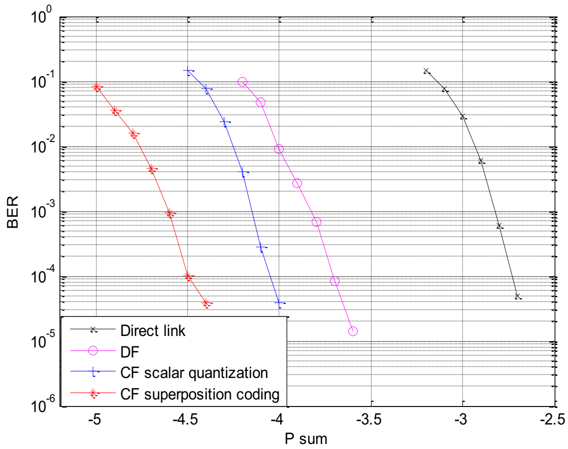

Figure 5. Bit error rate (BER) versus the total power of the system in AWGN channel.

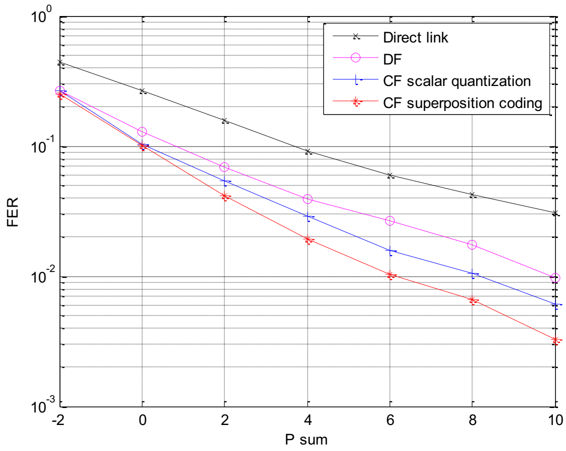

Figure 6. Frame error rate (FER) versus the total power of the system in block fading channel.

V. CONCLUSIONS

In this paper, we propose a practical CF scheme and coding strategy, based on the works about superposition coding for Wyner-Ziv problem. We apply the analysis of superposition coding for Wyner-Ziv problem to practical CF scheme design. From the simulation results, the proposed scheme outperforms the CF scheme using scalar quantization at the relay and the DF scheme when the relay-destination link is stronger than the source-relay link. Moreover, the analysis of the scheme on AWGN channel can be extended to the scenarios of fading channels that is more practical. The future work will be the optimization of the superposition codes for better performance.

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