Effect of the Connectivity on the Performance of Cooperative Network Coding

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***Abstract*— Cooperative Network Coding is a novel technology that synergistically integrates Network Coding with cluster-based Cooperative Communications to produce enhanced network reliability and security features, while improving the throughput. In this paper, we consider the effect of the connectivity on the performance of wireless sensor networks using Cooperative Network Coding, and we present scenarios where the values of the connectivity optimize the throughput. Generally, Cooperative Network Coding provides its optimal throughput when the number of nodes in the first cluster connected to the source node is equal to the number of source packets, the number of nodes in cluster i+1 connected to the node (i, j) is at least 4 and the destination node is connected to all the nodes in the last cluster. However, if for any reason the connectivity of nodes between two adjacent clusters is reduced, Cooperative Network Coding can increase its performance by connecting all the nodes in the first cluster to the source node.**

Keywords- Cooperative Network Coding; Cooperative Networks; Network Coding; Connectivity; Clustering.

I. INTRODUCTION

In *ad-hoc* wireless packet networks, such as sensor networks, a path (a sequence of nodes between the source and the destination) is chosen and then packets are forwarded along the path, as is shown in Fig 1. Because of the multiple hops that a packet generally makes to reach its destination, the probability of successful reception at the destination in a multihop network is generally lower than the probability of successful reception in a single hop. To overcome the link-level packet loss and to avoid significant end-to-end throughput degradation, networks often use link-level retransmissions. Moreover, if any packet is "lost" during the transmission, that specific packet is retransmitted from the source node. However, there is no guarantee that the retransmitted packet can be correctly received by the destination node.

To improve the probability of successful reception and throughput in multihop networks, the authors in [1] presented a novel technology know as *Cooperative Network Coding* that synergistically integrates Network Coding with Cooperative Communications to produce enhanced network reliability and security features, and which is expected to improve the throughput for a large class of networks, including wireless sensor networks, satellite networks, and selected military networks. The analysis of the performance of Cooperative Network Coding was conducted considering that this technology can be implemented without either link-level feedback or retransmissions. Then, to increase network reliability the authors in [2] considered Cooperative Network Coding with link-level retransmissions. Based on this analysis, and since there is a high probability that a node in the last cluster has a packet to transmit to the destination node, data-link feedback is implemented between nodes at the last cluster and the destination node.

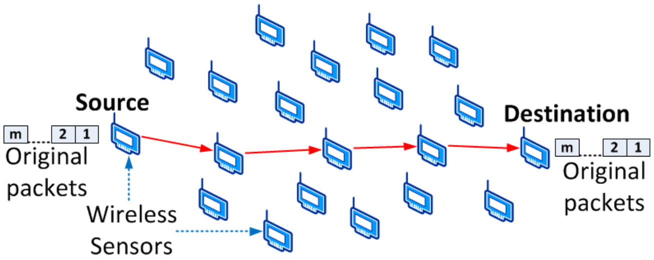

Figure 1. The Multihop Network model

In both [1] and [2], the metrics to evaluate the performance of Cooperative Network Coding are the probability of successful reception and the throughput. In this paper, we extend the work done in [1] by investigating the effect of the network's connectivity on the performance of Cooperative Network Coding.

The paper is organized as follows. In section II, we briefly summarize the work done in [1]. Section III presents the effect of the network's connectivity on Cooperative Network Coding. Finally, Section IV contains our conclusions.

II. RELATED WORK

Cluster-based Cooperative Communication with Network Coding in Wireless Networks was proposed by Haas and Chen in [1]. This novel technology, referred as Cooperative Network Coding, has the potential to significantly improve the communication capabilities, such as situation awareness. In addition, the information redundancy in Cooperative Network Coding improves reliability, as when some combination packets are in error, it is quite likely that other network paths have provided a sufficient number of correctly received combination packets for the destination node to recover the original packets. Moreover, Cooperative Network Coding increases throughput compared to multi-hop routing.

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Since the clusters can continuously change because some nodes can move away from the cluster or be disabled and other nodes can be incorporated to the cluster, Cooperative Network Coding incorporates the functions of route determination, creation and control of the clusters, and cluster-to-cluster transmission.

As opposed to traditional multihop networks, in Cooperative Network Coding nodes on a path (from a source to a destination) are replaced by clusters of nodes (Fig. 2), which are in geographically close proximity to each other. The i^{th} cluster contains n_i nodes.

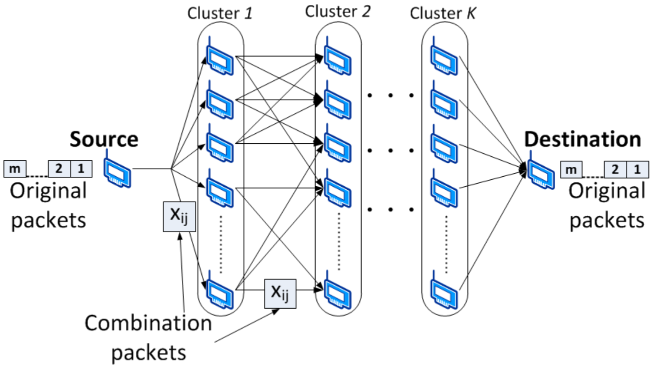


Figure 2. The Cooperative Network Coding model

The source creates n linear combinations of m original packets, where n must be at least the number of original packets, as shown in (1), where the j^{th} combination is given by

$$x_{Sj} = \sum_{l=1}^{m} c_{jl} x_l, \qquad (1)$$

where j goes from 1 to n, x_l are the original packets and the coefficients c_{ij} are randomly chosen from a Galois Field, $GF(2^q)$. Additionally, the multiplication and addition in (1) are also operations over the $GF(2^q)$. Then, the source transmits these linear combinations (referred as "combination packets") towards the nodes in the first cluster (Cluster *1*). Nodes in a cluster create new combination packets from the received combination packets x_p, as is shown in (2), and transmit those towards the next cluster.

$$x_{ij} = \sum_{p=1}^{q_{i-1}} c_{jp} x_p \qquad 1 \leq j \leq n_i \qquad (2)$$

where i is the number of cluster, j is the number of nodes in the i^{th} cluster and q_{i-1} is the number of combination packets received by node (i, j) from nodes in cluster $i - 1$.

Each node in a cluster (2 through K) acts as a MISO (Multiple Input, Single Output) node by receiving multiple combination packets and transmitting one new combination packet, as shown in Fig. 3.

The number of combination packets received by node (i, j) depends on the connectivity of the network, which is denoted as r. Cooperative Network Coding considers three metrics of connectivity. The first metric of connectivity is the number of nodes in the 1st cluster that are connected with the source node, r_s, which can vary from the number of original packets (m) to the number of nodes in the 1st cluster (n_1).

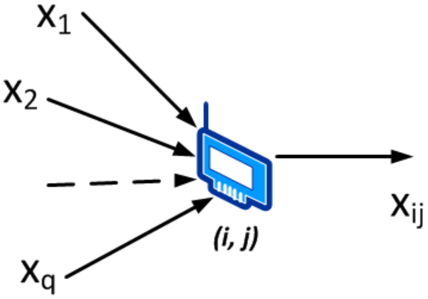


Figure 3. Node's Network Coding operation

For example, in Fig. 4, r_s is equal to 4. The second metric of connectivity is the number of nodes in cluster $(i + 1)$ that are connected with node (i, j), r_{ij}, which can vary from 2, because of the minimum value for cooperating, to the number of nodes in cluster $(i + 1)$. For example, in Fig. 4, r_{i3} is equal to 5. And, the last metric of connectivity is whether the node j in the last cluster is connected to the destination, r_{Kj}, which could be either 0 or 1. For example, in Fig. 4, r_{K1} is 0 and r_{K2} is 1.

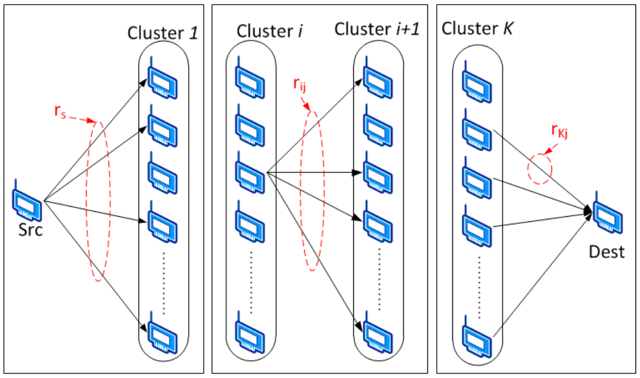


Figure 4. The Connectivity of Cooperative Network Coding

Based on these parameters of connectivity, the destination receives combination packets, from cluster K, and decodes the original message. The destination must receive at least m linearly independent packets to recover the m original packets.

In the next section, we analyze the effect of the connectivity on the performance of Cooperative Network Coding, through simulations.

III. EFFECT OF THE CONNECTIVITY ON THE PERFORMANCE OF COOPERATIVE NETWORK CODING

In this section we discuss various scenarios of the connectivity. The different scenarios indicate whether a significant improvement in the network's throughput is achieved or no throughput improvement at all.

As we can see in Fig. 4, the connectivity's metrics can take the following values:

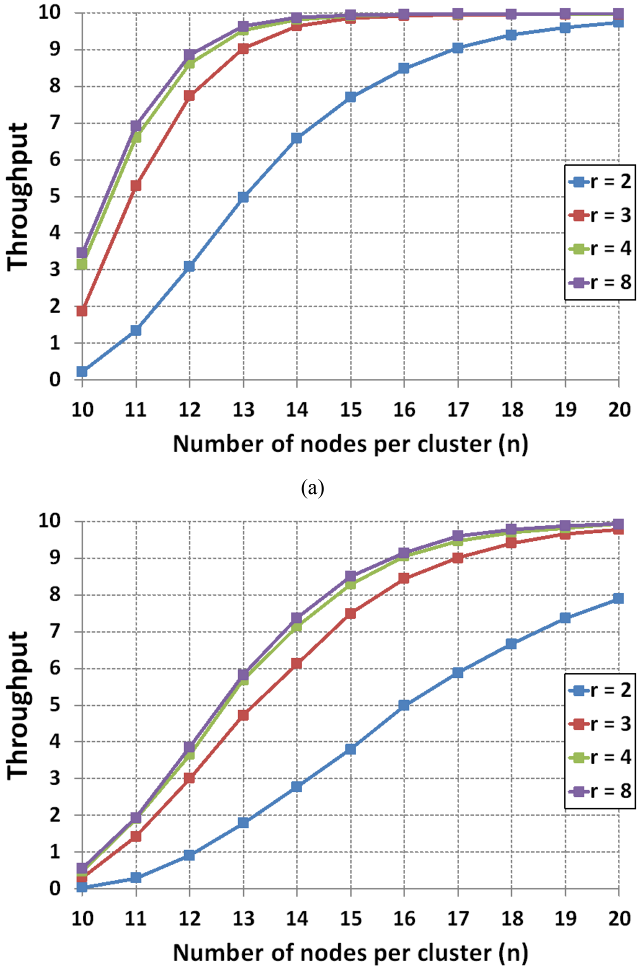
$$r_s \in [m, m + 1, \dots, n_1 - 1, n_1]$$

$$r_{ij} \in [2, 3, \dots, n_{i+1}-1, n_{i+1}]$$
$$r_{Kj} \in [0, 1]$$

where r_{ij} should be at least 2 to have cooperation among the nodes.

The results presented in this paper were obtained through simulations by running 10000 experiments and averaging the results. Also, we considered the following assumptions:

- The number of original packets m is 10,
- All the clusters have the same number of nodes $\mathrm{n} = \mathrm{n_i}$,
- There are 3 clusters between the source and destination nodes ($\mathrm{K} = 3$),
- The number of nodes in cluster 1 connected to the source is equal to the number of original packets ($\mathrm{r_s} = \mathrm{m} = 10$),
- The connectivity between node j in the cluster i and nodes in the cluster $\mathrm{i} + 1$ is the same for all the nodes between cluster 1 and cluster $\mathrm{K} - 1$ $\left(\mathrm{r} = \mathrm{r_{ij}}\right)$,

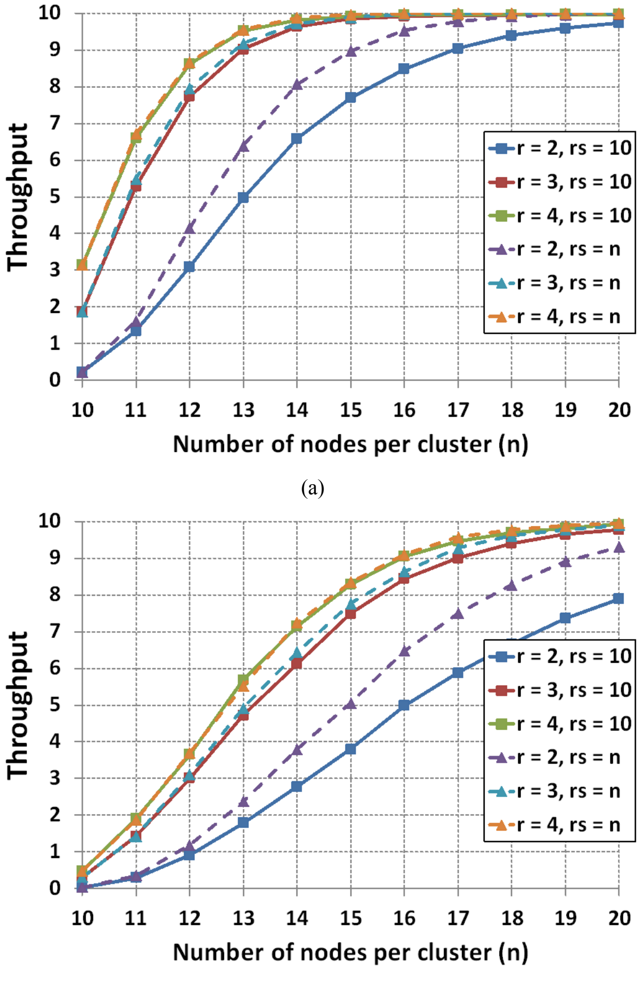


(a)

(b)

Figure 5. Throughput vs. number of nodes per cluster (n) for $r_s = m = 10$ and different values of r_{ij}: (a) the probability of transmission loss of a link (p) is 0.1, (b) the probability of transmission loss of a link (p) is 0.25

- The destination nodes is connected to all the nodes in the last cluster $\left(\mathrm{r_{Kj}} = 1, \forall\, \mathrm{j}\right)$,
- The probability of transmission loss of a link is the same for all the links $\mathrm{p} = \mathrm{p_{(i,j)(i+1,l)}}$.



(a)

(b)

Figure 6. Comparison of throughput for r_s equal m and r_s equal n for different values of connectivity (a) the probability of transmission loss of a link (p) is 0.1, (b) the probability of transmission loss of a link (p) is 0.25

Figure 5 shows the throughput vs. the number of nodes per cluster for different values of connectivity and probability of transmission loss of a link of 0.1 and 0.25. As we can see in Fig. 5, the throughput gain is minimum compared to the increase of cooperation among the nodes for values of connectivity greater than 4. Therefore, we concentrate our work on investigating the effect of the connectivity on the performance of Cooperative Network Coding for connectivity values r equal 2, 3 and 4, where $\mathrm{r} = 4$ is the optimal value for the connectivity of the nodes between two adjacent clusters $\left(\mathrm{r_{ij}}\right)$.

A comparison of the effect of the connectivity between the source and nodes in the first cluster, r_s, is presented in Fig. 6. As is shown, increasing the connectivity r_s provides a marginal improvement on the performance for connectivity values between nodes in cluster $i+1$ and the node (i,j), r_{ij}, is greater or equal than 3. When the connectivity between nodes in cluster $i+1$ and the node (i,j) is 2, we can obtain a significant increase of the performance of Cooperative Network Coding by connecting all the nodes in the first cluster to the source node.

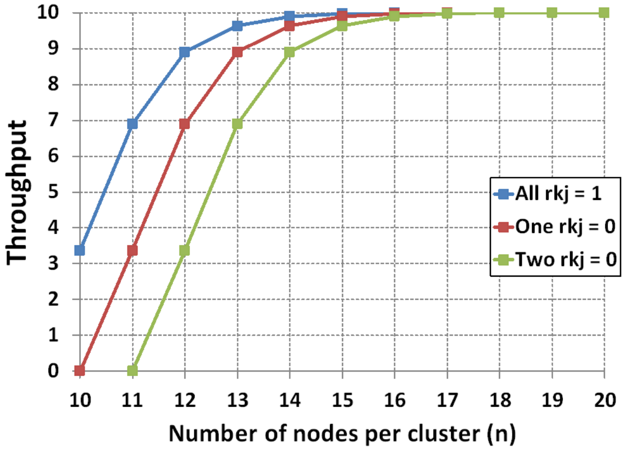


Figure 7. Effect of connectivity between nodes in the last cluster and the destination node for the probability of transmission loss of a link (p) of 0.1

Figure 7 shows the effect of the connectivity between nodes in the last cluster and the destination node. The connectivity r_{Kj} impacts the performance of Cooperative Network Coding, because when one node in the last cluster is disconnected from the destination, the performance of Cooperative Network Coding for a cluster size n is the same as the performance for a cluster size $n-1$ when all the nodes in the last cluster are connected to the destination. This connectivity is directly related to a node failure, because if a node in the last cluster fails, for any reason, its connectivity to the destination is set to be 0. A failure of a node in any cluster between the first and the penultimate clusters has little or no effect on the connectivity, so it does not affect the performance of the network.

IV. CONCLUSIONS

Our study in this paper focused on analyzing the effect of the connectivity on the performance of Cooperative Network Coding. Based on the range of parameters we have investigated, Cooperative Network Coding achieves its optimal performance when $\mathrm{r_s}$ is equal to m, $\mathrm{r_{ij}}$ is 4 and $\mathrm{r_{Kj}}$ is 1 for all the $j's$. Any increase on the connectivity, $\mathrm{r_s}$ and $\mathrm{r_{ij}}$, offers just marginal gain in throughput and introduces unnecessary redundant traffic in the network.

For connectivity $\mathrm{r_{ij}}$ equal to 2 and $\mathrm{r_{Kj}}$ equal to 1 for all the $j's$ and, by setting the connectivity $\mathrm{r_s}$ equal to the number of nodes per cluster m, Cooperative Network Coding can achieve an increase of throughput of about 34% and 37% for probabilities of transmission loss of a link of 0.1 and 0.25, respectively.

The connectivity $\mathrm{r_{Kj}}$ has a direct effect on the performance of Cooperative Network Coding because if the destination is disconnected from one of the nodes in the last cluster, the network performance is reduced and the throughput for a cluster size n is equal to the throughput of a cluster size $n-1$.

In conclusion, the optimal value of connectivity for Cooperative Network Coding to deliver the largest throughput is achieved by having all the nodes in the 1^{st} cluster connected to the source, the destination node connected to all the nodes in the last cluster and $\mathrm{r_{ij}}$ equal to 4. However, if the goal is to minimize the number of network coding operations per node, due to the constrains of processing capability that certain wireless sensor nodes have, an alternative would be to improve the network performance by connecting all the nodes in the first cluster to the source and connecting only two nodes of cluster $(i+1)$ to the node (i,j).

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