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A Clustering Algorithm That Maximizes Throughput in 5G Heterogeneous F-RAN Networks

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***Abstract*—In this paper, a clustering algorithm is proposed that dynamically determines the locations of fog nodes in 5G wireless networks, which are upgraded from small cells, in order to maximize throughput assuming the number of fog nodes and small cells are given as *a priori* information. The proposed algorithm dynamically clusters the small cells around the fog nodes. The approach is based on a soft clustering model where one small cell can be connected to many fog nodes. The numerical results demonstrate that the proposed clustering algorithm significantly enhances the throughput and lowers the latency with respect to the distance-based K-means hard clustering algorithm or Voronoi tessellation model.**

***Index Terms*—HetNets, clustering, fog computing.**

I. Introduction

Heterogeneous networks (HetNets) that have many small cells (SCs) overlaid in macrocells are of paramount interest to address the huge throughput increase [1]. The biggest challenge in HetNets is the interference stemming from the lack of coordination among SCs. This might be overcome by fog networking, which can control and coordinate the SCs to ensure better interference mitigation [2],[3]. Adapting the principles of fog computing by upgrading some SCs to fog nodes to solve the performance challenges in HetNets is a promising approach. However, there are many unsolved issues in the integration of fog computing with HetNets. Determination, or selection, of the fog nodes among many SCs and the connection of other SCs to the fog nodes, i.e., which SCs are served by which fog nodes, are open issues that are worth studying.

The simplest approach is to employ the classic Voronoi tessellation model so that SCs are clustered according to their distances and the leader of each cluster, i.e., cluster-head becomes fog node and each SC is assigned to the closest fog point, which corresponds to the K-means clustering algorithm in machine learning [4]. However, this algorithm does not guarantee a high throughput, because the fog node at the closest distance may have the poorest channel for that SC. The primary goal of this paper is to increase the overall data rate (i.e., throughput) by proposing a novel clustering algorithm to meet the various requirements in 5G that can be employed for both *downlink* and *uplink*. It is worth noting that the higher the data rate the smaller the transmission delay and the smaller the queueing delay, because the service rate of the queues is directly proportional to the data rate, which, in turn, can ensure more stable queues in the SCs. Hence, maximizing the data rate, or throughput, will minimize latency when all parameters are kept constant.

Fog or edge computing has been extensively studied to provide location-aware services to the mobile end users with higher data rate and lower latency, e.g., see [5]-[7] and references therein. However, selecting the locations of the fog nodes among many SCs, whose locations are all known, and clustering the SCs around fog nodes so as to enable improved service has not yet been considered[1]. Moreover, the widely used clustering algorithms in machine learning such as K-means clustering algorithm [4], *fuzzy clustering-means* (FCM) algorithm [9], *possibilistic c-means* (PCM) clustering algorithm [10] or hybrid approaches that combine FCM and PCM algorithms [11], [12] do not optimize clustering so as to maximize the throughput. The proposed soft or fuzzy clustering algorithm in this paper clusters the system with the ultimate aim of maximizing throughput.

In this paper, a novel soft clustering algorithm is proposed for a fog Radio Access Network (F-RAN) based on a well-known Water-Filling algorithm, but with a very different application perspective. Heretofore, the general aim in Water-Filling algorithm is to find the optimum transmission power levels for multi-user communication [13]-[16]. This paper employs the Water-Filling algorithm for clustering to determine the probability of connection between SCs and many fog nodes so that one SC can be connected to many fog nodes with a probability between 0 and 1, whose sum is equal to 1. The overall contributions of this paper when it comes to the adaptation of fog networks and machine learning to the HetNets are summarized as below:

- Dynamically determining, or selecting, the fog nodes among many SCs according to the channel conditions. To illustrate, a new set of fog nodes are determined for quasi-static fading channel for each channel change. Note that fog nodes are upgraded from SCs, likely via a SW upgrade.
- Cluster the SCs around fog nodes to maximize the data rate.
- Adapt the well-known Water-Filling algorithm in communication theory to machine learning to develop a novel soft clustering algorithm. To the best of the authors knowledge, this is the first study that proposes to use

[1]This idea is briefly mentioned in [8].

Water-Filling to cluster many data points around cluster-heads.

This paper is organized as follows. The system model and problem formulation are given in Section II, and a novel clustering algorithm is discussed in Section III, whose numerical results are presented in Section IV. The complexity of the proposed clustering algorithm is analyzed in Section V. The paper ends with the concluding remarks in Section VI.

II. SYSTEM MODEL AND PROBLEM FORMULATION

The primary advantage of increasing throughput with the HetNet architecture can be corrupted due to interference among SCs. Fog nodes can control SCs and alleviate interference in addition to providing them cloud-like functions such as communication, computation and storage services. Therefore, upgrading some SCs to fog nodes ensures coordination and decreases the interference level. There will be many fog nodes that will be upgraded from SCs within the area of interest, whose numbers are given as *a priori* information. How to determine the number of fog nodes and the optimum number are other problems [17], and are out of scope for this paper.

In the considered network model shown in Fig. 1, fog nodes are able to serve many SCs and one SC might access many fog units. In this model, the high power node (HPN) provides ubiquitous services, and SCs are stationary, i.e., they have fixed locations, and their locations are known. The locations of SCs that will be upgraded to fog nodes are found in this setting according to the quality of the channels. Accordingly, the location of fog nodes changes from one channel realization to another assuming that the channels among SCs are a quasi-static fading channel that changes slowly.

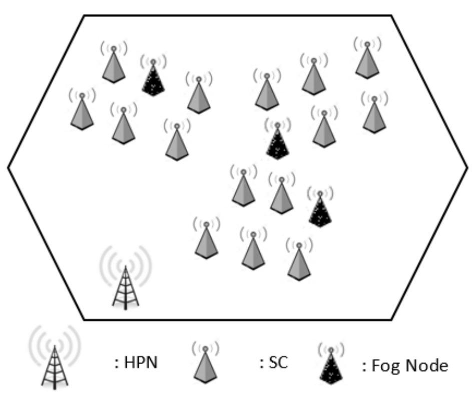

Fig. 1. A generic cellular network model with many SCs and fog nodes.

The major challenges for this network model is to dynamically determine the locations of fog nodes that will be upgraded from SCs for a quasi-static fading channel and specify which SCs can be controlled by or take services from which fogs. To illustrate, whether SCs should be connected to only one fog node or more than one fog node, the decision criterion to make a connection with fog nodes are among the aforementioned questions that this paper will address. To give clear-cut answers for these questions, a grouping or clustering algorithm has to be specified based on a similarity measure. Note that clustering the SCs around the fog units not only specifies the connection among them but also enables the SCs to leverage the limited resources more efficiently. Furthermore, this approach contributes to the virtualization of the network, which is one of the most desired feature in next generation 5G wireless networks.

The most important point in grouping or clustering is to determine a similarity criterion that reflects the strength of the relationship between two data points so that similar data is grouped together within a cluster. In this paper, the similarity measure is the quality of the wireless channels. A widely used similarity criterion based on the distance is where SCs are connected to the closest fog points, which is known as a Voronoi tessellation based on K-means clustering approach, cannot be a reasonable approach when it comes to addressing the high data rate and low latency requirements of 5G networks. It is clear that such an approach can give very poor performance when the closest distances have the worst fading channels, because this degrades the data rate and thus the transmission delay.

The clustering problem is formulated by considering the geographical locations of SCs and fog nodes as cluster members and cluster-heads, which are the leaders of each cluster, respectively. More specifically, suppose that there are N SCs and K fog nodes that represent cluster-heads in a 2-dimensional Euclidean space. Accordingly, a group of SCs can transmit to a group of fog nodes at the same time, and a successive interference cancellation (SIC) receiver may be utilized in the fog units to separate the signals coming from different SCs. Of course, this is an ideal case, so that interference is perfectly mitigated. Clusters are formed for this scenario to maximize the data rate or throughput while maintaining stable transmission queues based on a novel fuzzy or soft clustering algorithm whose details are given in the subsequent section.

III. A NOVEL FUZZY CLUSTERING ALGORITHM

Assume that we start with a total of $N + K$ SCs and K of them will be upgraded to fog nodes to control and provide services to the other N SCs. Note that the values of N and K are given as *a priori* information. Let $X = \{x_1, x_2, \cdots, x_N\} \in R^2$ be the locations of SCs, and $E = \{e_1, e_2, \cdots, e_K\} \in R^2$ represent the fog locations. That is, X and E denote the members of clusters and the centers of clusters, respectively. The general objective function that is to be optimized for a clustering algorithm can be stated as

$$J = \sum_{n=1}^{N} \sum_{k=1}^{K} F(\gamma_{nk} f(x_n, e_k)) \tag{1}$$

where γ_{nk} is the degree of membership that quantifies the similarity criterion so that $\gamma_{nk} \in \{0, 1\}$ for hard clustering and $\gamma_{nk} \in [0, 1]$ for fuzzy or soft clustering. $F(\gamma_{nk} f(x_n, e_k))$ is the general function that determines the clusters by linearly multiplying γ_{nk} with $f(x_n, e_k)$, which measures the similarity of any data point x_n for $n = 1, 2, \cdots, N$ with a cluster-head e_k for $k = 1, 2, \cdots, K$. These functions can be expressed for the conventional Euclidean distance based clustering as

$$f(x_n, e_k) = ||x_n - e_k||^2 \tag{2}$$

and

$$F(\gamma_{nk} f(x_n, e_k)) = \gamma_{nk} ||x_n - e_k||^2. \tag{3}$$

However, minimizing the distance does not necessarily minimize the total latency and the data rate, which is a more dominant factor in determination of latency with respect to propagation distance.

In our proposed fuzzy clustering algorithm, the clusters are formed in an attempt to maximize the data rate while ensuring the stability of queues in the SCs, and thus $f(.)$ and $F(.)$ are determined accordingly. In this direction, $f(.)$ represents the channel power as follows

$$f(x_n, e_k) = h_{nk} h_{nk}^* \tag{4}$$

where h_{nk} is the channel between x_n and e_k resulting in

$$F(\gamma_{nk} f(x_n, e_k)) = F(\gamma_{nk} h_{nk} h_{nk}^*). \tag{5}$$

Since one SC can connect to k number of fog nodes, (5) is defined as the weighted capacity between one SC and its connected fog node

$$F(\gamma_{nk} f(x_n, e_k)) = w_{nk} \sum_{j=1}^{k} \log(1 + \gamma_{nj} h_{nj} h_{nj}^*) \tag{6}$$

where w_{nk}'s are the weights determined by the fog or edge units to satisfy the stability of queues in the SCs by adjusting the service rate between each SC and fog, and the second term in the right-hand side provides the sum of communication rates when one SC is connected to k fog nodes for normalized bandwidth and transmission power. Fog nodes must monitor the queuing buffers of the SCs while giving services; otherwise unstable queues may occur leading to infinite latency. In fact, this is the reason of involving w_{nk}'s in the objective function (12), which can guarantee the stability of queues in the SCs. Let q_t^n be the queue size of the n^{th} SC that has an arrival rate of α_t^n and a service rate of u_t^n. The queue dynamics can then be written as

$$q_{t+1}^n = \max(q_t^n - u_t^n, 0) + \alpha_t^n. \tag{7}$$

A queue becomes strongly stable if

$$\lim_{T \to \infty} \sup \frac{1}{T} \sum_{t=1}^{T} E(q_t^n) < \infty. \tag{8}$$

This factor is taken into account in the optimization formulation so that w_{nk}'s are adjusted regarding u_t^n to ensure stability.

The degree of membership γ_{nk} is set up based on *a posteriori* probability $p(e_k|x_n)$ that specifies the connection of n^{th} SC with the k^{th} fog unit such that $\gamma_{nk} = p(e_k|x_n)$ and

$$\sum_{k=1}^{K} p(e_k|x_n) = 1. \tag{9}$$

The connection of N number of SCs with K number of fog nodes produces a $N \times K$ clustering matrix $\mathbf{\Gamma}$ whose entries are γ_{nk} for $n = 1, 2, \cdots, N$ and $k = 1, 2, \cdots, K$. The value of each γ_{nk} lies in between 0 and 1, and the sum value of each row in $\mathbf{\Gamma}$ is equal to 1. To optimize the values of γ_{nk}, the following objective function based upon (6) is utilized given by

$$J_n = \sum_{k=1}^{K} w_{nk} \sum_{j=1}^{k} \log(1 + \gamma_{nj} h_{nj} h_{nj}^*) \tag{10}$$

which can be written for N SCs as

$$J = \sum_{n=1}^{N} J_n. \tag{11}$$

Since SCs are independently located within the area of interest and interference is assumed to be mitigated with a SIC receiver, maximizing (10) will also maximize (11) and the optimization problem can then be formulated considering the fact that (8) can be satisfied with a proper u_t^n as

$$\max \sum_{k=1}^{K} w_{nk} \sum_{j=1}^{k} \log(1 + \gamma_{nj} h_{nj} h_{nj}^*) \tag{12}$$

subject to

$$\forall w_{nk} \geq 0, \; k = 1, 2, \cdots, K \tag{13}$$

$$w_{nk} = c_k u_t^n, \; k = 1, 2, \cdots, K \tag{14}$$

$$c_k \geq 0, \; k = 1, 2, \cdots, K \tag{15}$$

$$\forall \gamma_{nk} \geq 0, \; k = 1, 2, \cdots, K \tag{16}$$

$$\sum_{k=1}^{K} \gamma_{nk} = 1. \tag{17}$$

The Lagrangian function can then be given by (18) whose equality to satisfy the Karush-Kuhn-Tucker (KKT) condition produces

$$\frac{\partial L(\gamma_n, \rho, r, z, v, \mu)}{\partial \gamma_{nk}} = \frac{\partial J_n}{\partial \gamma_{nk}} + v_k - \mu = 0. \tag{19}$$

Considering the v_k as a slack variable [13], (19) becomes

$$\sum_{j=k}^{K} \frac{w_{nj} |h_{nk}|^2}{1 + \sum_{i=1}^{j} \gamma_{ni} |h_{ni}|^2} = \mu. \tag{20}$$

To further simplify (20), let

$$\sigma_{kj} = \frac{1 + \sum_{i=1, k \neq j}^{j} \gamma_{ni} |h_{ni}|^2}{|h_{nk}|^2} \tag{21}$$

that yields

$$\sum_{j=k}^{K} \frac{w_{nj}}{\sigma_{kj} + \gamma_{nk}} = \mu. \tag{22}$$

$$L(\gamma_n, \rho, r, z, v, \mu) = J_n + \sum_k \rho_k w_{nk} + \sum_k r_k(w_{nk} - c_k u_t^n) + \sum_k z_k c_k + \sum_k v_k \gamma_{nk} - \mu(\sum_k \gamma_{nk} - 1) \quad (18)$$

It is analytically intractable, i.e., there is no closed-form expression, to solve (22), and a variant of the Water-Filling algorithm [14], hereafter labeled as edge location assisted Water-Filling (ELA-WF), is employed to find the exact values of γ_{nk}. Accordingly, the water level μ^* satisfying (17) is first found then γ_{nk} for $k = 1, 2, \cdots, K$ is determined according to μ^*. Note that solving (22) for all $n = 1, 2, \cdots, N$ gives the $N \times K$ clustering matrix $\mathbf{\Gamma}$. The index of each row for $n = 1, 2, \cdots, N$ in $\mathbf{\Gamma}$ indicates one specific SC location. Similarly, the index of each column for $k = 1, 2, \cdots, K$ in $\mathbf{\Gamma}$ shows the location of fog nodes. Notice that it is assumed that all the locations of SCs are known at the beginning, which is reasonable due to static nature of SC locations, and thus there is a one-to-one mapping between the locations of SCs and fog nodes, and the rows and columns of matrix $\mathbf{\Gamma}$.

The aim of the proposed clustering algorithm is to first determine the locations of SCs that will be upgraded to fog nodes among many SCs placed in a given area, which are static and whose locations are known, and then cluster the remaining SCs around these selected fog nodes. Clustering will occur in a probabilistic way so that each SC is connected to all upgraded fog nodes with a certain probability, which is found according to the quality of channel. The details of ELA-WF are given below in Algorithm 1:

Algorithm-1: Edge Location Assisted Water-Filling (ELA-WF)

1) Initialize K and N to the number of fog nodes and SCs respectively given as *a priori* information.
2) Set $N = N + K$ and $K = N$.
3) Set the entries of a $(N+K) \times (N+K)$ channel matrix $\boldsymbol{H}$ to the channel values between all SCs and fog nodes. Notice that all diagonal terms of $\boldsymbol{H}$ should be 0.
4) Initialize $\gamma_{nk}^{(0)} = 0$ for $k = 1, 2, \cdots, K$ and $n = 1, 2, \cdots, N$ corresponding to each entry in $\boldsymbol{H}$. This yields an $(N+K) \times (N+K)$ matrix $\mathbf{\Gamma} = 0$.
5) Iterate starting from $l = 1$
   a) Initialize $n = 1$.
   b) Select $w_{nk}^{(l)}$ that satisfies (14) for $\forall k$ except $k = n$.
   c) Find $\sigma_{kj}^{(l)}$ for $\gamma_{nk}^{(l-1)}$ using (21) for $\forall k$ except $k = n$.
   d) Apply Water-Filling algorithm to find $\gamma_{nk}^{(l)}$ using (22) for $\forall k$ except $k = n$.
   e) Update $\gamma_{nk}^{(l)} = 1/K * \gamma_{nk}^{(l)} + (1 - 1/K) * \gamma_{nk}^{(l-1)}$ for $\forall k$ except $k = n$ to ensure the convergence as usually done in Water-Filling algorithm, see [15] and [16].
   f) Set $\gamma_{nk}^{(l)} = 0$ for $k = n$.
   g) Repeat steps *b* to *f* for $n = 2, \cdots, N$.
   h) Continue iterations until $\gamma_{nk}^{(l)} - \gamma_{nk}^{(l-1)} < \epsilon$ for $\forall k$ and $\forall n$.
6) Find the expected value of γ_{nk} with respect to n for $k = 1, 2, \cdots, K$ as $\mathbf{v} = [E[\gamma_{n1}]E[\gamma_{n2}] \cdots E[\gamma_{nK}]]$.
7) Reset N and K to the initial values that are given as *a priori* information.
8) Determine the index of the highest K values in $\mathbf{v}$. Note that these indices give the locations of the fog nodes that will be upgraded from SCs.
9) Remove the K rows that correspond to the indices of the K highest values in $\mathbf{v}$. This reduces the dimension of matrix $\mathbf{\Gamma}$ to $N \times (N + K)$.
10) Remove the N columns of $\mathbf{\Gamma}$ that correspond to the indices of the lower N locations in $\mathbf{v}$, which produces a $N \times K$ matrix of $\mathbf{\Gamma}$.
11) Normalize the sum of the entries of γ_{nk} to 1 in each row for $k = 1, 2, \cdots, K$, which shows the connection probability of each SC to the K number of fog nodes.

The convergence of this algorithm can be easily justified using the results of [14]-[16] that utilizes an iterative Water-Filling algorithm for MIMO channel and employs SIC at the receiver. Although the proposed algorithm has a centralized nature, this does not lead to significant computational complexity increase owing to the limited number of SCs within the area of interest. Note that each fog network covers a local area [2], and thus there are a limited number of SCs within the area of interest.

IV. Numerical Results

The proposed ELA-WF soft clustering algorithm is compared to Voronoi tessellation model, which corresponds to the connection of a SC with the fog node that has the strongest signal for the average values of channels. First, the comparison is made regarding data rate for normalized bandwidth or spectral efficiency (SE) assuming that there are 4 fog nodes and 100 SCs. In the ELA-WF algorithm, each SC will probabilistically be connected to all fog nodes depending on the channel conditions without any distance concern. On the other hand, Voronoi tessellation only considers the smallest distance so that each SC will be connected to the closest fog unit without considering the channel quality. To observe the difference between these soft and hard clustering algorithms, the spectral efficiencies are given in Fig. 2 in terms of signal-to-noise ratio (SNR). There is a significant advantage of ELA-WF algorithm compared to Voronoi tessellation model, e.g., more than a 2 dB advantage is observed for the spectral efficiency of 1 bits/s/Hz.

The same experiment is repeated by increasing the number of fog units from $K = 4$ to $K = 8$ for 100 SCs. Here, it is worth noting that the performance of both ELA-WF and Voronoi tessellation model decrease as depicted in Fig. 3. For the former case, when there are more fog nodes under the constraint of (17), the assigned power level to the best quality channel lowers contingent upon Water-Filling, which

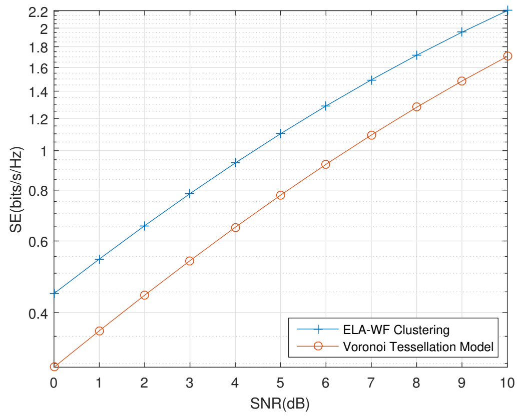

Fig. 2. Spectral efficiency of the proposed algorithm compared to the Voronoi tessellation for $K = 4$

decreases the spectral efficiency. The reduction for the latter case can be understood because the more fog nodes the more likely the SC will be connected to the poor channel, which implies a diminishment in the spectral efficiency.

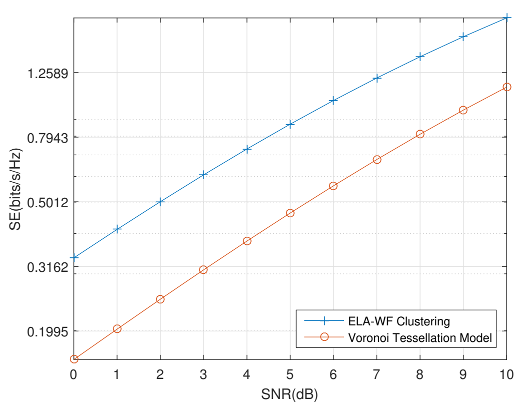

Fig. 3. Spectral efficiency of the proposed algorithm compared to the Voronoi tessellation in terms of spectral efficiency for $K = 8$

Another performance metric for the comparison of ELA-WF algorithm with respect to the-state-of-art Voronoi tessellation is the end-to-end latency that is composed of many components. It is a difficult task to completely represent the end-to-end latency while transmitting packets from an SC to the fog node composed of transmission delay, propagation delay, queueing delay and processing delay. Therefore, the latency characterization in this paper has been simplified and mainly focused on the transmission and queueing delay, since ELA-WF algorithm minimizes the transmission delay while ensuring a bounded queueing delay. Indeed, propagation delay can be ignored, because the fog nodes are situated close to the SCs, and the speed of light for a few kilometers can only lead to a delay on the order of microseconds. Moreover, processing delays can be greatly reduced with SDN/NFV networking [18]. As a result, transmission and queueing delays are the major sources of end-to-end latency for future wireless networks, and thus the end-to-end latency are presented for ELA-WF clustering algorithm, while ignoring the propagation and processing delay. The performance of ELA-WF clustering algorithm is compared with the Voronoi tessellation model assuming that queueing delay is bounded although this model does not guarantee it.

To give quantitative results about latency, some assumptions are made so that the data arrival rates for each SC are taken as 1 kbps, and each transmitted packet is assumed to be 1024 megabits. Moreover, there are 4 fogs and 100 SCs within the area of interest without any loss of generality. The result is depicted in Fig. 4 for a bandwidth of 100 MHz, which clearly shows the superiority of the ELA-WF clustering over the Voronoi tessellation model in terms of SNR. When the same

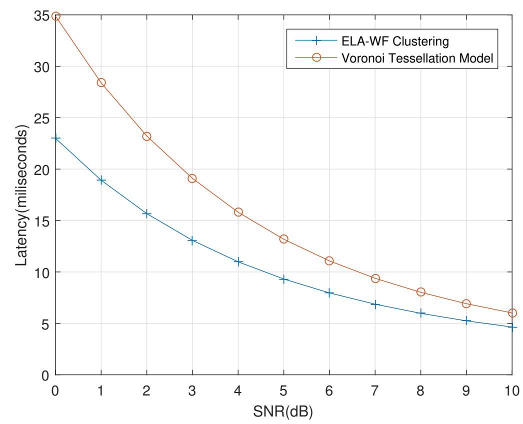

Fig. 4. Latency of the proposed algorithm compared to the Voronoi tessellation in terms of SNR for $K = 4$

simulation is made for $K = 8$, an interesting result is observed in Fig. 5. Accordingly, the latency is not affected much for ELA-WF as the number of fog units increases, whereas the latency in Voronoi tessellation increases considerably. It can be deducted that the proposed clustering not only increases the data rate but also provides more robust latency for higher numbers of fog units, which is likely in 5G networks

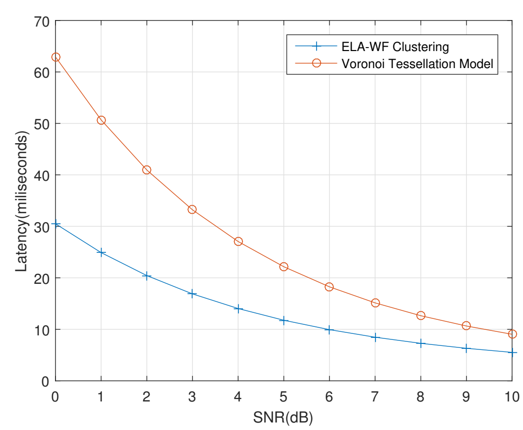

Fig. 5. Latency of the proposed algorithm compared to the Voronoi tessellation as in terms of SNR for $K = 8$

To further elaborate on the proposed clustering algorithm, SNR is fixed at 5 dB without any loss of generality and the impact of bandwidth is investigated by comparing the performance of the ELA-WF clustering and the Voronoi tessellation model in Fig. 6 regarding latency. As can be seen, ELA-WF clustering has a significant latency advantage, especially for low bandwidths. Furthermore, the 1 ms latency requirement in 5G can be achieved for 1 GHz bandwidth at 5dB with ELA-WF algorithm when the processing delay is ignored, which can be obtained for a smaller bandwidth when the SNR is higher than 5dB.

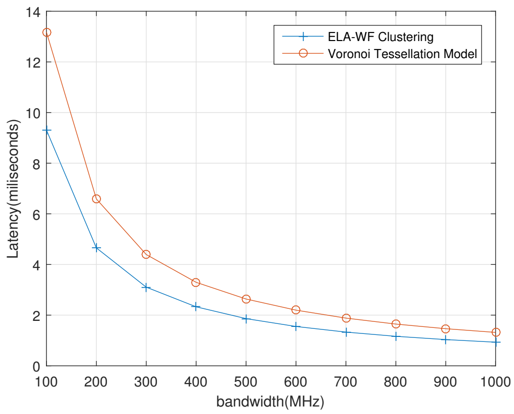

Fig. 6. Latency of the proposed algorithm compared to the Voronoi tessellation in terms of bandwidth for $K = 4$

V. Complexity Analysis

The proposed ELA-WF clustering algorithm is composed of many stages from initialization to finding relevant parameters for intermediate steps and applying the Water-Filling algorithm, etc. A step-by-step complexity analysis of ELA-WF is given in this Section. Accordingly, the complexity of steps $(1)-(4)$ in the proposed ELA-WF algorithm is equal to $O(1)$, because the initialization and settings can be performed independent of the number nodes. On the other hand, step (5) requires some computation that is scaled with N bringing $O(N)$ complexity due to steps $(5b), (5c), (5e)$. In addition, Water-Filling based on the binary search such as bisection algorithm in step $(5d)$ results in $O(\log(N))$. Repeating all these steps for all N in $(5g)$ finally produces a complexity of $O(N^2)$. The complexity of remaining steps from (6) to (11) is less than $O(N^2)$, and hence the overall complexity of ELA-WF becomes $O(N^2)$.

In case of the complexity analysis of K-means clustering, 2 major steps are in concern. In the first step, the points are clustered for fixed cluster-heads that brings a complexity of $O(N)$. In the second step, the cluster-heads are optimized according to the given points that results in a complexity of $O(N)$ for a general dissimilarity measure. Hence, the overall complexity becomes $O(N^2)$ [4]. Notice that this complexity is the same with the proposed ELA-WF soft clustering.

VI. Conclusions

An important problem is addressed in this paper, specifying the connection of SCs with many possible fog nodes by a novel clustering algorithm, which aims to maximize the throughput by increasing the total communication rate based on a Water-Filling algorithm, while ensuring stable queueing delays. The comparison of ELA-WF with the state-of-the-art Voronoi tessellation model, which does not guarantee a stable queueing delay, clearly demonstrates the superiority of the proposed ELA-WF clustering algorithm in terms of substantially increasing the throughput, or spectral efficiency in parallel with reducing latency. Indeed, the advantage of the proposed clustering algorithm grows over the classically used model in terms of the spectral efficiency and latency when the number of fog nodes are increased. This result is quite important and useful in the design of next generation wireless networks that will have fog or edge computing units.

References

[1] A. Damnjanovi, J. Montojo, Y. Wei, T. Ji, T. Luo, M. Vajapeyam, T. Yoo, O. Song, and D. Malladi, "A survey on 3gpp heterogeneous networks", *IEEE Wireless Communications*, vol. 18, no. 3, pp. 10-21, June 2011.

[2] F. Bonomi et al., "Fog Computing and its Role in the Internet of Things", *Proc. Wksp. Mobile Cloud Computing*, Helsinki, Finland, pp. 1316, Aug. 2012.

[3] M. Chiang and T. Zhang, "Fog and IoT: An overview of research opportunities", *IEEE Internet Things J.*, vol. PP, no. 99, pp. 11, 2016.

[4] C. M. Bishop, Pattern Recognition and Machine Learning, Springer, Aug. 2006.

[5] S. Wang, et.al, "A Survey on Mobile Edge Networks: Convergence of Computing, Caching and Communications", *IEEE Access*, vol.5, pp. 6757-6779, March 2017.

[6] S. Yi, C. Li, Q. Li, "A survey of fog computing: Concepts applications and issues", *Proceedings of the 2015 Workshop on Mobile Big Data*, 2015.

[7] L. M. Vaquero, L. Rodero-Merino, "Finding your way in the fog: Towards a comprehensive definition of fog computing", *ACM SIGCOMM CCR*, 2014.

[8] E. Balevi and R. D. Gitlin, "Unsupervised Machine Learning in 5G Networks for Low Latency Communications", *IEEE International Performance Computing and Communications Conference (IPCCC)*, 2017.

[9] J. C. Bezdek, Pattern Recognition With Fuzzy Objective Function Algorithms, New York: Plenum, 1981.

[10] R. Krishnapuram and J. Keller, "A possibilistic approach to clustering", *IEEE Trans. Fuzzy Syst.*, vol. 1, no. 2, pp. 98110, Apr. 1993.

[11] N. R. Pal, K. Pal, and J. C. Bezdek, "A mixed c-means clustering model", *in IEEE Int. Conf. Fuzzy Systems*, pp. 1121, 1997.

[12] N. R. Pal, K. Pal, J. M. Keller, and J. C. Bezdek, "A possibilistic fuzzy c-means clustering algorithm", *IEEE Trans. Fuzzy Syst.*, vol. 13, pp. 517-530, Aug. 2005.

[13] S. Boyd and L. Vandenberghe, Convex Optimization., 2004.

[14] W. Yu, W. Rhee, S. Boyd, and J. Cioffi, "Iterative water-filling for Gaussian vector multiple-access channels", *IEEE Trans. Inf. Theory*, vol. 50, no. 1, pp. 145152, Jan. 2004.

[15] N. Jindal, W. Rhee, S. Vishwanath, S. A. Jafar, and A. Goldsmith, "Sum power iterative waterfilling for multi-antenna Gaussian broadcast channels", *IEEE Trans. Inf. Theory*, vol. 51, pp. 15701580, Apr. 2005.

[16] M. Kobayashi, G. Caire, "An iterative water-filling algorithm for maximum weighted sum-rate of gaussian MIMO-BC", *IEEE J. Select. Areas Commun.*, vol. 24, no. 8, pp. 1640-1646, Aug. 2006.

[17] E. Balevi and R. D. Gitlin, "Optimizing the Number of Fog Nodes for Cloud-Fog-Thing Networks", *submitted for publication*. See https://arxiv.org/pdf/1801.00831v1.pdf

[18] N. McKeown et al., "OpenFlow: Enabling innovation in campus networks", *SIGCOMM Comput. Commun. Rev.*, vol. 38, no. 2, pp.69-74, Mar.2008.