Improving the Performance of OFDM-Based Vehicular Systems through Diversity Coding

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Abstract: **In this paper, we present diversity coded orthogonal frequency division multiplexing (DC-OFDM), an approach to maximize the probability of successful reception and increase the reliability of OFDM-based systems through diversity coding. We focus on the application of DC-OFDM to vehicular networks based on IEEE 802.11p technology and analyze the performance improvement using this new technology.**

It is shown that DC-OFDM significantly improves the performance of vehicular *ad hoc* networks in terms of throughput and the expected number of correctly received symbols.

Index Terms: **Diversity coding (DC), orthogonal frequency division multiplexing (OFDM), performance, probability of success, reliability, symbol error probability.**

I. INTRODUCTION

Vehicular communications play an important role in vehicle safety and transportation efficiency. The objective of vehicular communication is to ensure vehicle safety for the drivers and passengers and to reduce time and fuel consumption, among other services. A few of the primary applications envisioned for vehicular networks are emergency notifications for automotive safety, notification, and prevention of vehicles during collision, location-based information and vehicle tracking services, high-speed tolling, real-time traffic updates, and Internet access with multimedia streaming.

For short and medium range communications, wireless access in vehicular environments (WAVE)/IEEE 802.11p systems have been devised using the wireless local area network (WLAN) technologies in these systems. These systems have decent transmission range and power, although, they are limited in terms of coverage distance. WAVE systems are complex as the vehicular environment is dynamic. Therefore, it is important to maintain a stable and reliable wireless connection for a significant period of time. Moreover, WAVE, which is a wireless scheme that provides vehicle-to-vehicle (V2V) communication and vehicle-to-infrastructure (V2I) communication, has as its primary application in providing emergency safety measures for vehicles.

IEEE 802.11p uses orthogonal frequency division multiplexing (OFDM) [1] to transmit information. OFDM is a widely used technology in fourth generation wireless networks (4G) and 802.11a/g/n WLANs that achieve high transmission rates over dispersive channels by transmitting serial information through multiple parallel carriers. The transmission bandwidth is divided into many narrow sub-channels, which are transmitted in parallel, such that the fading each channel experiences is flat.

Many applications need to be communicated in a timely manner, so reliability of these networks is a concern. To overcome these issues, the novel idea of employing diversity coding (DC) [2] across OFDM subchannels is proposed. DC-OFDM based systems are capable of achieving better spectrum efficiency with excellent transmission rates, improved throughput, perform better during multipath fading, and retrieve lost information easily without the need of retransmission or feedback from the transmitter when compared to other similar schemes employed in vehicular *ad hoc* networks (VANETs). Using DC, reliable information can be transmitted especially for time-critical applications even when a reliable infrastructure is available.

The paper is organized as follows. In Section II, we summarize the prior work on wireless access in vehicular environments, OFDM and DC. Section III presents a literature review of the related work. Our approach, DC-OFDM is presented in Section IV, including an analysis of the system performance. Section V presents results of the performance of DC-OFDM. Finally, in Section VI, we present our conclusions and future research directions.

II. LITERATURE REVIEW

A. Wireless Access in Vehicular Environments

WAVE is a combination of both IEEE 802.11p and IEEE 1609.x working in the dedicated short range communications (DSRC) band, especially for both the physical (PHY) and the medium access control (MAC) layers.

The IEEE 802.11p standard PHY layer is similar to the 802.11a standard, but with more specific requirements to meet the communication requirements between vehicles or between a vehicle and the infrastructure in current vehicular environments. The key points that drove the 802.11p standard are the relative speed (distances) between the vehicles, the maximum possible coverage distance ($\sim$1000 meter radius), varying multipath channel effects in multiple environments and most importantly, the reliability and the security of the message broadcasted in the network. The 5.85–5.925 GHz band was chosen to minimize the interference and overcrowding present in the operating bandwidth of 802.11a WLANs. The 75 MHz bandwidth of 802.11p is divided into seven 10 MHz channels each with a 5 MHz margin at the lower end. Since the channel bandwidth is halved (as compared to 802.11a channel bandwidth of 20 MHz), the data rates, carrier spacing, and other parameters are also be halved. While doubling the symbol duration (8 μs) helps prevent inter-

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symbol interference (ISI), it also helps in reducing the effects of the multipath channel in rural, urban and sub-urban environments under study.

B. Orthogonal Frequency Division Multiplexing

The OFDM is a technique that transmits a serial high data rate stream in parallel over multiple subcarriers with much lower data rates per subcarrier.

Similar to the IEEE 802.11a architecture, the IEEE 802.11p has a total of 64 carriers that are divided into 48 subcarriers to carry the data, 4 pilot carriers to make the signal robust against frequency offset, and the remaining 12 subcarriers are null. The subcarriers subjected to inverse fast Fourier transform (IFFT) convert from the frequency domain to the time domain. The output of the IFFT process is the OFDM symbol. To this symbol, guard intervals (GIs) are added to prevent the symbol from ISI and inter-carrier interference (ICI). For high-speed scenarios, the channel is difficult to predict and there is an increase in the Doppler shift, which in turn degrades the quality of the signal and increases the bit error rate (BER). The choice of having a longer guard interval is a tradeoff between the bandwidth of the data and the reliability as longer GI reduces the throughput of the channel. In the case of IEEE 802.11p standard, the guard interval is observed to be longer than the maximum excess delay in all the environments. The minimum transmission rate for IEEE 802.11p is 3 Mbps for BPSK modulation with 1/2 coding rate up to a maximum of 27 Mbps for 64 quadrature amplitude modulation (QAM) with 3/4 coding rate.

The BER of an OFDM-based system for M-QAM modulation scheme in additive white gaussian noise (AWGN) and Rayleigh channels, respectively, are given by [3]:

$$P_e = \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{E_b}{N_0}\frac{6\log_2 M}{M^2-1}}\right), \quad (1)$$

$$P_e = \frac{M-1}{M\log_2 M}\left(1-\sqrt{\frac{\frac{E_b}{N_0}\frac{3\log_2 M}{M^2-1}}{\frac{E_b}{N_0}\frac{3\log_2 M}{M^2-1}+1}}\right) \quad (2)$$

where M is the order of the modulation scheme and E_b/N_0 is the energy per bit to noise power spectral density ratio. The symbol error probability (P_{ser}) for a M-QAM and M-phase-shift keying (PSK) modulated OFDM signals, respectively, under an AWGN channel are given by [4]:

$$P_{\text{ser}} = 4\left(1-\frac{1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{E_s}{N_0}\frac{3}{M-1}}\right) - 4\left(1-\frac{1}{\sqrt{M}}\right)^2 Q^2\left(\sqrt{\frac{E_s}{N_0}\frac{3}{M-1}}\right), \quad (3)$$

$$P_{\text{ser}} = \frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}} e^{\left(-\frac{E_s}{N_0}\frac{\sin^2\frac{\pi}{M}}{\sin^2\theta}\right)} d\theta. \quad (4)$$

If we assume that the channel is an ideal linear time-invariant frequency non-dispersive AWGN channel, the receiver sees the OFDM signal as a group of parallel AWGN channels with equal signal-to-noise ratio (SNR) that has a similar performance as a single carrier system. That is, the average symbol error probability for an OFDM symbol is equal to the symbol error probability of a subcarrier.

$$P_{\text{ser}} = P_{\text{ser}_{\text{OFDM}}}, \quad P_{\text{ser}} = \frac{1}{N}\sum_{i=1}^{N} P_{\text{ser}_{\text{SubCarrier}_i}}. \quad (5)$$

Since $P_{\text{ser}_{\text{SubCarrier}_i}} = P_{\text{ser}_{\text{SubCarrier}}}, \forall i$

$$P_{\text{ser}} = P_{\text{ser}_{\text{SubCarrier}}}. \quad (6)$$

However, the OFDM signal has a lower SNR compared to the single carrier signal due to the cyclic prefix.

When a multipath channel is assumed (e.g., Rayleigh distributed), the symbol error probability for a quadrature PSK (QPSK) modulated OFDM signal can be calculated as [5]:

$$P_{\text{ser}} = \frac{3}{4} - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\left(1-\frac{1}{\pi}\tan^{-1}\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right) \quad (7)$$

where

$$\gamma = \frac{E_b}{\frac{\pi^2 f_d^2}{3\Delta f^2}E_b + N_0}. \quad (8)$$

Δf is the subcarrier spacing, f_d is the maximum Doppler shift and is calculated as:

$$f_d = \frac{v f_c}{c} \quad (9)$$

where v is the relative speed between transmitter and receiver, and c is the speed of wave (typically the speed of light in the vacuum). The probability of having δ symbol errors in an OFDM symbol can be calculated using the binomial probability mass function:

$$P_\delta = \binom{n}{\delta}(1-P_{\text{ser}})^{n-\delta} {P_{\text{ser}}}^{\delta}. \quad (10)$$

And the probability of having no symbol errors $(\delta = 0)$ in an n-subcarrier OFDM symbol can be calculated using (10):

$$P_{\delta=0} = (1-P_{\text{ser}})^n. \quad (11)$$

The probability of symbol error of a coded OFDM system, using block coding (n, k, t), is given by [5]:

$$P_{\text{ser}_{\text{coded}}} = \frac{1}{n}\sum_{\delta=t+1}^{n}\delta P_\delta. \quad (12)$$

By implementing DC in vehicular communications that use OFDM-based technologies, such as IEEE 802.11p systems, the bandwidth is utilized in an efficient manner, the reliability of the communication is improved and lost information can be recovered in different vehicular communication scenarios. For real-time traffic applications, such as emergency response, link failures cannot be acceptable as the applications are delay sensitive. Both feedback and rerouting add to the delay and it is preferable to use DC technique, a spatial diversity technique, to recover the lost information.

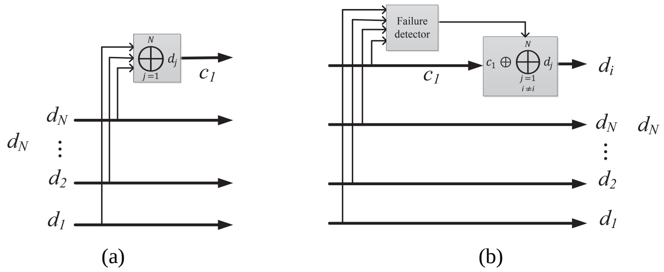


Fig. 1. System with $1-for-N$ DC: (a) Transmitter and (b) receiver.

C. Diversity Coding

DC [2], [6], [7] is a feed-forward spatial diversity technology that enables near instant self-healing and fault-tolerance in the presence of wireless link failures. The protection paths (c_i) carry information that is the combination of the uncoded data lines (d_i). Fig. 1 shows a DC system that uses a spatial parity check code for a point-to-point system with N data lines and 1 protection line. If any of the data lines fail (e.g., d_3), through the protection line (c_i), the destination (receiver) can recover the information of the data line that was lost (d_3) by taking the mod 2 sum of all of the received signals.

A network is transparently self-healed when any combination of N coded links survive among M diverse links. This technique is very efficient without the necessity of having the packets reroute in other directions.

Assuming there are N links in the network, d_j lines to be the information carrying bits in the binary form and there is an extra line present to protect the network from fading or other failures.

$$c_1 = d_1 \oplus d_2 \oplus \cdots \oplus d_N, \quad (13)$$

$$c_1 = \bigoplus_{j=1}^{N} d_j \quad (14)$$

where $\oplus$ represents the XOR function and the extra line $N+1$ carries the checksum c_1. If any one of the lines between 1 and N fails, then the receiver detects the line/channel with failure and obtains $\hat{d}_i$:

$$\hat{d}_i = c_1 \oplus \bigoplus_{\substack{j=1 \\ j \neq i}}^{N} d_j. \quad (15)$$

According to (15), the estimated unknown variable $\hat{d}_i$ can be calculated from the logical XOR function performed on all the d_j variables from 1 to N, except d_i. After expanding c_1, (14), it is easy to obtain the information of the failed i^{th} line as $d_j \oplus d_j = 0$ and $\hat{d}_i = d_i$.

$$\hat{d}_i = \bigoplus_{j=1}^{N} d_j \oplus \bigoplus_{\substack{j=1 \\ j \neq i}}^{N} d_j. \quad (16)$$

By using just one extra line, the lost information in the failed link can be recovered instantaneously without rerouting or providing a feedback channel to the transmitter.

This concept can be extended to multiple line failures and also to recover lost packets in packet-based networks. The delay in a network changes whenever there is a link failure and when recovery is needed, otherwise, the delay in a normal operating network is constant. The delay occurs because the system contains different links, each having different lengths, with each link causing delay based on the distance between source node and destination node. However, an OFDM-based system such as 802.11p perfectly fits in the DC concept because some subcarriers can transport the data and other subcarriers (a few) can transport the protection information.

The system with $1-for-N$ DC can be generalized to a $M-for-N$ DC, where the coded information is calculated as [2]

$$c_i = \sum_{j=1}^{N} \beta_{ij} d_j, \quad i \in \{1, 2, \cdots, M\} \quad (17)$$

where c_i and d_j are protection (diversity coded) and data (uncoded) symbols, respectively. The β coefficients are given by:

$$\beta_{ij} = \alpha^{(i-1)(j-1)} \quad (18)$$

where α is a primitive element of a Galois field $GF(2^q)$, $i = \{1, 2, \cdots, M\}$ and $j = \{1, 2, \cdots, N\}$. The total number of transmitted symbols is equal to the number of data symbols plus the number of protection symbols $(N+M)$, where the number of protection symbols is typically less than the number of data symbols $(M \leq N)$. So, the β coefficients form a Vandermonde matrix:

$$[\beta_{ij}] = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \alpha & \alpha^2 & \dots & \alpha^{(N-1)} \\ 1 & \alpha^2 & \alpha^4 & \dots & \alpha^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha^{M-1} & \alpha^{(M-1)2} & \dots & \alpha^{(M-1)(N-1)} \end{bmatrix}. \quad (19)$$

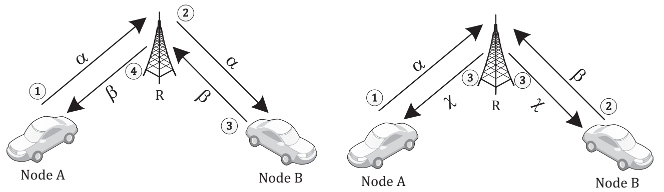
Current approaches use network coding (NC) [8], a successor of DC, to improve the performance of OFDM-based systems such as the case of IEEE 802.11p. These approaches are applied at packet, symbol or signal levels. The following section presents these approaches.

III. RELATED WORK - NETWORK CODING FOR OFDM-BASED SYSTEMS

NC, which is an enhancement of DC, is a concept where packets are combined and transmitted through different nodes or locations. The aim of NC is to reduce the number of packet retransmission and thereby improve system bandwidth and throughput. There are two different NC techniques widely used at different levels and are: 1) Symbol level and 2) packet level, applicable to both PHY and MAC layers in IEEE 802.11p (WAVE) systems.

A. Network Coding in the MAC Layer

NC in the MAC layer has evolved a lot over the years and in this paper, the relationship between NC and a MAC protocol for IEEE 802.11 is studied. Although the NC scheme can



(a) (b)

Fig. 2. Wireless multihop systems with and without NC: (a) Non-WNC multihop system and (b) WNC multihop system [9], [10].

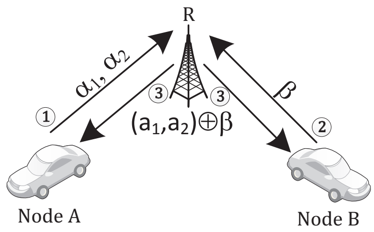


Fig. 3. RDNC [16].

be used to receive the packets, NC coupled with an acknowledgement will be a powerful technique in helping the receiving node know about the overhead packets and has high probability of successful deliveries [9].

The wireless NC (WNC) scheme is typically a MAC layer oriented scheme based on the proposed method of sending acknowledgment (ACK) packets from both the nodes A and B for OFDM based systems. This technique complies with the IEEE 802.11a (WLAN) standard. As shown in Fig. 2(b), both A and B transmit their packet to the relay R which stores them and performs XOR operation to the packets and sends the resultant output to A and B. Since A and B know the packet they transmitted, they decode the XORed output and obtain the necessary information. Instead of sending the packets through four transmissions (time slots) to reach both A and B (see Fig. 2(a)), using wireless NC reduces it to three transmissions. This improves the throughput of the system and the bandwidth.

Nodes A and B send their ACK packets in the form of direct and delayed signals and the relay R demodulates them as one ACK packet. Generally, the OFDM technique has some subcarriers that have redundant information that is useful in letting the relay know if A and B have sent their respective ACK packets.

The WNC scheme is found to have high efficiency and very low packet transmission loss compared to traditional schemes. Transmission control protocol (TCP) throughput is observed to increase by 3.4 Mbps at 25 dB SNR and has much better packet loss rate (PLR) performance which ensures higher reliability than in conventional systems [10].

B. Network Coding in the PHY Layer

In the PHY layer, symbol level NC is a predominant method and there are different approaches used in V2V and V2I communications [11]– [15]. Vehicular communications are point-to-point communications with an infrastructure such as roadside units. However, in reality, most of the communications take place in *ad hoc* network mode VANET with the help of relays. The vehicles adopt the *ad hoc* network mode in situations when there is no availability of roadside units in the particular area of interest. There are several approaches that implement NC in the PHY layer. A few of them are:

B.1 Rate Diverse Network Coding

This approach combines modulation techniques with the conventional NC scheme in the physical layer. Rate diverse NC (RDNC) [16] was proposed for IEEE 802.11a/g networks to demodulate the signal received at different nodes according to their respective channel types.

For example, as shown in Fig. 3, because of the channel conditions, node A sends two packets to the relay node (R) using binary PSK (BPSK) and node B sends only one packet to the relay using QPSK. The relay codes both packets from node A with one packet from node B and transmits it to node B using QPSK modulation. Since QPSK has 2 bits per symbol, the modulated packet will be able to pair with the two BPSK packets transmitted from node A.

This scheme is still valid even if the packets to be XORed do not have the same bits. Among the coding rates that comply with the IEEE 802.11a standard, RDNC uses $3/4$ coding rate as it is capable of carrying more information than the other rates. Therefore, it is limited to 9 Mbps (BPSK), 18 Mbps (QPSK), and 36 Mbps (16 QAM). When different symbols carrying different bits and modulation techniques are sent, there is a problem in combining these into one single packet and then sending it to the destination nodes. BPSK is the preferred modulation method in many conventional systems for simplicity, but it is not efficient not to use higher-order modulation techniques for rate links.

B.2 Zero Cost Retransmission

Zero cost retransmission (ZCR) [17] is a physical layer symbol-level NC scheme that piggybacks a new packet while retransmitting another packet. Upon receiving the retransmitted packet, the piggybacked packet is demodulated using NC coupled with a modulation scheme. The piggybacked packet can be demodulated depending on the successful reception of the retransmitted packet. The retransmitted packet is successfully received with the help of maximal ratio combining method used with the previous corrupted packet. That is, consider that one node has sent α and the packet is corrupted upon reception at the destination node (α'). So, the node retransmits by encoding α and β and decoding α first and then decoding β (if α is correctly received).

The advantage of this scheme is that two packets are received in the receiver section during just one retransmission thereby reducing the overhead. The encoding and decoding cannot be performed in the conventional method of using bitwise XOR. Instead, symbol level physical layer NC coupled with a modulation technique is used to map the α and β information to the constellation points according to the used modulation technique.

B.3 CodePlay

CodePlay [18] is a symbol level NC (SLNC) technique for live multimedia streaming service in VANETs. SLNC is a type of NC applied to a smaller group of consecutive bits within a packet. Live multimedia streaming (LMS) service is used for real-time applications such as live video streaming which is useful for intelligent navigation and also for non-real time applications like video on-demand. LMS is generally employed in conventional wired or wireless networks where the link is stable and reliable. When the LMS scheme is used in VANETs, it will lead to severe packet loss due to the varying effects of the channel and the bandwidth utilization is inefficient. The objectives of this scheme are: 1) Better utilization of the bandwidth, 2) reliable service delivery ensuring all near-by users have the same delays, and 3) to provide smooth playback having a high and stable streaming rate.

However, the main difference between these schemes and our approach is that NC is typically applied to two hop communication through a relay and for multicasting, where it has been proven that NC provides throughput gain [19]– [23]. Our approach focuses on point-to-point communications and takes advantage of the spatial (frequency) transmission of the information when using OFDM. Moreover, our approach can be used by itself, or it can be used along with any other forward error correction technique at bit level such as convolutional codes or Reed-Solomon codes.

IV. DIVERSITY CODED ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (DC-OFDM)

OFDM [1], [24] is a well-known technology used for 4G and 802.11 systems to achieve high data rates by transmitting the information through several orthogonal subcarriers such as IEEE 802.11p that provides wireless access in vehicular environments. DC-OFDM is a novel coding technique, which can be applied to extent 802.11p, and operates at the symbol level to transmit information through orthogonal frequencies, as in OFDM schemes, to enhance the performance and increase the reliability of the communication. DC-OFDM increases the reliability of these point-to-point OFDM wireless connections by transmitting data in some set of subcarriers and protection data (redundant or coded information) through another subset of carriers.

DC-OFDM is based on the observation that in OFDM communications the information is transmitted through orthogonal frequencies (parallel channels) and each subchannel can experience different channel effects. The DC-OFDM communication is shown in Fig. 4.

This novel technique of applying coding across OFDM carriers differs from the traditional coded OFDM where channel coding techniques, such as convolutional codes, Reed-Solomon codes, in combination with interleaving are used in each subcarrier channel in the time domain.

A. System Model

Combining DC with OFDM promises high reliability in vehicular, and other, communications while preserving high transmission rates. This is achieved by transmitting coded information across the OFDM carriers. That is, most of the carriers transport original information while the remaining (few) carriers transport coded information. The coded information is the result of the combination of the original information as in DC. As shown in Fig. 5, if any of the carriers that transport data (d_1, d_2, up to d_N) is lost because of a fade or because of the number of errors in a carrier exceeds the error correction capability of the forward error correction code (FEC), the information from the lost carrier can be recovered from the (received) protection carriers (c_1 for this simple example).

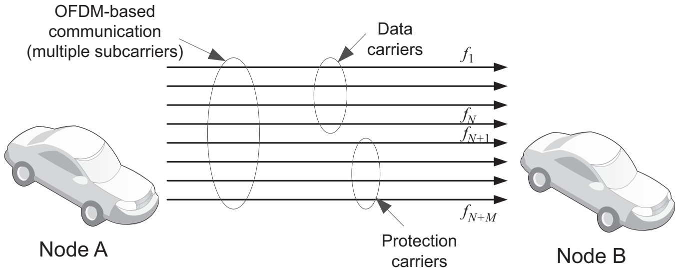


Fig. 4. DC-OFDM communication system.

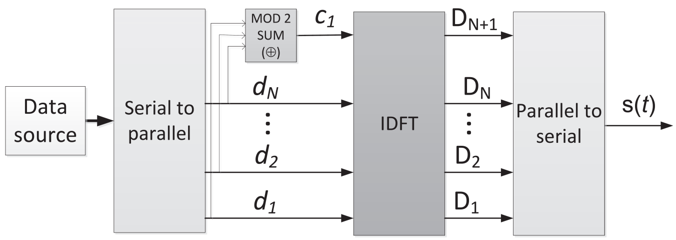


Fig. 5. System with $1-for-N$ DC-OFDM communication links (transmitter).

The information transported by the protection subcarrier c_1, as shown in Fig. 5, is calculated using (14). That is:

$$c_1 = \bigoplus_{j=1}^{N} d_j. \tag{20}$$

At the receiver, the decoding process is carried out using (16). If there is no failure in the data lines (data subcarriers), the information transmitted through the protection subcarrier is discarded. However, if there is a link failure in any of the data subcarriers, a failure detection algorithm detects the failure and informs the receiver which data subcarrier should be omitted and the information of the data subcarrier with failure is recovered with the information provided by the protection subcarrier. That is, if the information of the data subcarrier d_i is lost or corrupted, it can be recovered using c_i, as shown in Fig. 6.

From (16), we have:

$$\hat{d}_i = \bigoplus_{j=1}^{N} d_j \oplus \bigoplus_{\substack{j=1 \\ j\neq i}}^{N} d_j. \tag{21}$$

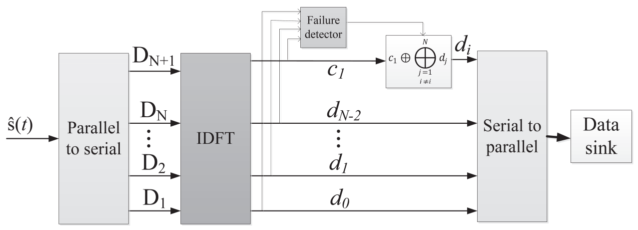


Fig. 6. System with $1 - for - N$ DC-OFDM communication links (receiver).

Expanding (21), we have:

$$\hat{d}_i = (d_1 \oplus \cdots \oplus d_i \oplus \cdots d_N) \oplus (d_1 \oplus \cdots \oplus d_{i-1} \oplus d_{i+1} \oplus \cdots \oplus d_N). \quad (22)$$

Given that $d_j \oplus d_j = 0$, (22) becomes:

$$\hat{d}_i = d_i. \quad (23)$$

By using just one subcarrier to transmit the coded information, the lost information in the failed link can be instantaneously recovered. Assuming that the probability of link error (p_i) is the same for all the links/subcarriers $(p = p_i, \forall i)$ the probability of successfully receive the correct information through at least any x links, out of the N data lines plus 1 protection line $(N + 1)$, is calculated as:

$$Ps = \text{Prob}(x \geq 1), \quad (24)$$

$$Ps = \sum_{t=1}^{N-1} \left(\frac{\prod_{i=1}^{t} i}{(N+1)^t} \right) \binom{N+1}{t} (1-p)^t p^{N+1-t} + \sum_{t=1}^{N-1} \binom{N+1}{t} (1-p)^t p^{N+1-t}. \quad (25)$$

However, since the region of interest is when the information has been correctly received through at least N links, (26) is reduced to:

$$Ps = \text{Prob}(x \geq N), \quad (26)$$

$$Ps = \sum_{t=1}^{N-1} \binom{N+1}{t} (1-p)^t p^{N+1-t}. \quad (27)$$

As shown in Fig. 5 and (20), each link can carry as few as one bit to implement a 1 for N DC system, because with one bit we can calculate a Galois Field of up to two elements $\{0, 1\}$, $GF(2^1)$. In other words, the number of protection links is limited by the number of bits per link. That is, the larger the number of bits to be transmitted by each link, the larger the number of protection links that can be implemented. This is because the number of protection links (subcarriers) is limited to the Galois Field $[GF(2^q)]$ size q to calculate the information that is transmitted through the protection links. If we would like to relate the number of protection links and the modulation schemes in an OFDM system directly, we can see that only high order modulation schemes can be used with DC because of DC spatial transmission characteristic. Table 1 shows the parameters for a DC OFDM-based system that take into account the IEEE 802.11p standard. That is, the number of subcarriers is 48, the number of data bits per OFDM symbol depends on the modulation scheme, and the code rate depends on the data rate. As we mentioned before, a maximum of one protection subcarrier can be created for a BPSK modulation scheme, maximum 3 protection links can be created for QPSK modulation scheme, 15 protection subcarriers can be created for 16 QAM, and 24 protection links can be created for 64 QAM. In other words, only 16 QAM with coding rate of ¾, 64 QAM with coding rate of ⅔, and 64 QAM with coding rate of ¾ would be suitable to directly create the spatial protection trough DC while maintaining the same structure as the IEEE 802.11p standard.

The probability of successful reception at the destination for DC-OFDM-based systems that use 16 QAM ¾, 64 QAM ⅔, or 64 QAM ¾ modulation schemes can be calculated using (27).

Nevertheless, since we are interested in studying the effects of DC in OFDM-based schemes, regardless of the modulation scheme and FFT size, in the following subsection, we present how DC works for any modulation scheme and FFT size.

B. Operation of DC - OFDM

DC, which is a spatial feed-forward error correction technique, is well suited to work on OFDM-based systems because the protection "lines" can be transmitted through some of the subcarriers. Since the number of protection subcarriers depends on the Galois Field size q, we first assign q bits per subcarrier in the serial to parallel conversion, as shown in Fig. 7. The number of bits to be transmitted per subcarrier is calculated based on the number of data and protection subcarriers, N and M, respectively and is given by [2]:

$$q \geq \lceil \log_2(N + M + 1) \rceil. \quad (28)$$

The total number of data plus protection lines (subcarriers) should be at most equal to the FFT size because the number of subcarriers is limited to the FFT size:

$$M + N \geq \text{FFT}_{\text{size}}. \quad (29)$$

The protection information that is transmitted through some of the OFDM subcarriers is calculated using (17):

$$c_i = \sum_{j=1}^{N} \beta_{ij} d_j, \quad i \in \{1, 2, \cdots, M\}. \quad (30)$$

In formulating DC-OFDM, we followed the Vandermonde matrix to create the coded symbols, which are linearly independent of each other, and each coded symbol (c_i) has the information of all the data symbols (d_j). Of course, we could have followed an approach such as using Hamming, or other, codes where each coded symbol (parity bits) has the information of only some data bits in different combinations. However, in this case, the receiver would need additional processing. By randomly choosing the β coefficients, linearly independency of the coded symbols is not guaranteed.

Fig. 8 shows the DC at the source node, where each subcarrier carriers a symbol of q bits. Moreover, the information transmitted through each subcarrier (data or protection subcarrier) is

Table 1. DC as a function of the modulation scheme for IEEE 802.11p.

Modulation	Code rate (R)	Coded bits per subcarrier (N_{BPSC})	Coded bits per OFDM symbol (N_{CBPC})	Data bits per subcarrier (N_{DBPS})	Data rate (Mbps)	Carriers with data	Max GF(2^{bits})	Max total carriers
BPSK	1/2	1	48	24	3	24	1	25
	3/4	1	48	24	4.5	36	1	37
QPSK	1/2	2	96	48	6	24	3	27
	3/4	2	96	72	9	36	3	39
16 QAM	1/2	4	192	96	12	24	15	39
	3/4	4	192	144	18	36	15	51
64 QAM	2/3	6	288	192	24	32	24	56
	3/4	6	288	216	27	36	24	60

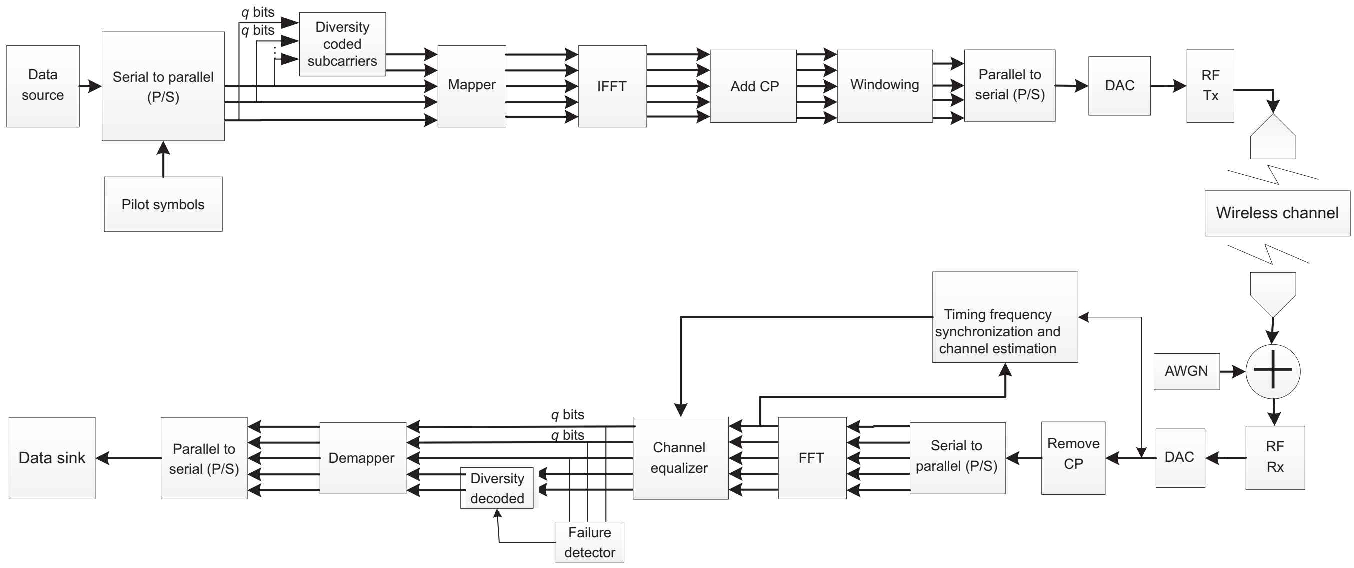


Fig. 7. DC-OFDM block diagram.

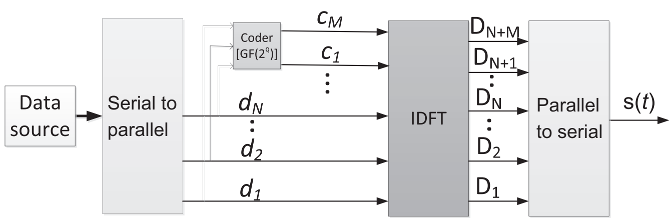


Fig. 8. $M-for-N$ DC-OFDM communication links.

predetermined and known by the transmitter and receiver. That is, the subcarrier index (location) is predefined for each subcarrier to transport either data or protection information.

Since the information transmitted through the data lines (subcarriers) is uncoded, the coding coefficients of the data lines form an identity matrix of size N (I_N) as shown below:

$$\mathrm{Subcarrier}_j \left\{ \overbrace{\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}}^{\mathrm{Dataline}_j} \right. \tag{31}$$

where $j \in \{1, 2, \cdots, N\}$. The coefficients of the protection lines are formed by the β_{ij} coefficients matrix, (19):

$$\mathrm{Subcarrier}_{i+N} \left\{ \overbrace{\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \alpha & \alpha^2 & \dots & \alpha^{(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha^{M-1} & \alpha^{(M-1)2} & \dots & \alpha^{(M-1)(N-1)} \end{bmatrix}}^{\mathrm{Dataline}_j} \right. . \tag{32}$$

The assignment of the data and protection lines to each subcarrier is predefined to minimize the computational complexity in both transmitter and receiver. The assignment can be sequential, where the data lines $\{1, 2, \cdots, N\}$ can be assigned to the N first subcarriers and the protection lines $\{1, 2, \cdots, M\}$ can be assigned to the next M subcarriers $\{N+1, N+2, \cdots, N+M\}$, or it can be interleaved, where for example, the first data line is assigned to the first subcarrier, the first protection lines is assigned to the second subcarrier, the second data line is assigned to the third subcarrier, and so on. This will depend on the DC rate. We define the DC code rate as the ratio of the number of data lines (subcarriers) to the number of data plus protection lines (subcarriers) ratio:

$$\mathrm{DCcoderate} = \frac{N}{N+M}, \tag{33}$$

At the receiver, the coefficients of the data and protection lines form the following matrix, which depends on the information that was correctly received at the destination:

$$\boldsymbol{\beta'} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 1 & 1 & 1 & \dots & 1 \\ 1 & \alpha & \alpha^2 & \dots & \alpha^{(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha^{M-1} & \alpha^{(M-1)2} & \dots & \alpha^{(M-1)(N-1)} \end{bmatrix}. \tag{34}$$

The receiver, by using the $\boldsymbol{\beta'}$ matrix coefficients, a $(N+M)-$ by $-$ N matrix, can find the transmitted data by recovering the lost information in the data lines through the protection lines. That is, the receiver uses only N rows out of the $N+M$ rows from the $\boldsymbol{\beta'_N}$ matrix coefficients to recover the information of the data lines:

$$\boldsymbol{\beta'_N x} = \boldsymbol{b_N}. \tag{35}$$

The receiver preferably uses as many indexes of the data lines as possible to faster decode the information that is lost during transmission. In other words, the receiver uses as many elements of the identity matrix, (31), as the implementation will allow. If no data line is lost during transmission, no decoding process is needed at the receiver and the information transmitted through the protection lines is discarded. The vector formed by the data lines $\boldsymbol{x}$ is:

$$\boldsymbol{x} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_N \end{bmatrix} \tag{36}$$

and $\boldsymbol{b_N}$ is the vector formed by the correctly received information at the destination with the same indexes as the $\boldsymbol{\beta'_N}$ matrix.

The receiver can recover the lost information transmitted through the data lines by performing Gaussian elimination of the $\boldsymbol{\beta}$ coefficients (protection lines). This is a fast process because some of the row elements of the coefficients matrix are already in the row canonical form.

Assuming that the probability of link error (p_i) is the same for all the links $(p = p_i, \forall i)$ the probability of successfully receiving the correct information through at least any N links, out of the N data lines plus M protection lines, is calculated as:

$$Ps = \text{Prob}(x \geq N), \tag{37}$$

$$Ps = \sum_{t=N}^{N+M} \binom{N+M}{t} (1-p)^t p^{N+M-t}. \tag{38}$$

However, the assumption that all the links have the same probability of link error may be unrealistic, because in an OFDM system each subcarrier can experience different channel effects. A general formula to calculate the probability of successfully receiving the correct information through at least any N links, out of the N data lines plus M protection lines is:

$$Ps = \sum_{t=N}^{N+M} \left[\sum_{\vec{a} \in A} \left(\prod_{i \in a_1} \prod_{j \in a_0} p_i (1-p_j) \right) \right] \tag{39}$$

where:

- A is a set of $N+M$ binary sequences of all the 2^{N+M} possible combinations. A binary sequence can contain either 0 or 1, where "1" means that the transmission was successful and "0" otherwise. The number of $1-s$ in A is t and the number of $0-s$ is $(N+M-t)$; so there are $\binom{N+M}{t}$ such sequences.
- $\vec{a}$ is a particular sequence from the set A, a_0 is a set of all indices j of $\vec{a}$ such that $a(j) = 0$, and a_1 is a set of all indices i of $\vec{a}$ such that $a(i) = 1$. Thus, $\|a_0\| + \|a_1\| = N+M$.
- p_i is the probability that the information transmitted through subcarrier i is correctly received at the destination.

The following section presents our results for our DC-OFDM scheme for different parameters as the number of data and protection lines, DC code rates, among others.

V. RESULTS

In this section, we discuss the performance of DC-OFDM as measured by the probability of successful reception at the destination. We have analyzed the effect of different parameters, such as: Number of data links, number of coded (protection) links, modulation technique, and DC code rate, to optimize the communication's probability of successful reception of an OFDM symbol at the receiver. Moreover, we have compared our approach (DC-OFDM) to existing OFDM-based systems, such as IEEE 802.11p, that do not use coding in the spatial domain (across sub-channels). Extant OFDM-based systems were described in subsection II-B and the DC-OFDM approach was presented in Section IV.

First, we start comparing the performance of $1-for-N$ DC-OFDM system to extant OFDM systems that do not use DC (no DC). Fig. 9 shows the probability of successfully receiving at the destination the information of N data links (subcarriers) as a function of the symbol error rate per subcarrier $(p_{\text{ser}_i} = p_{\text{ser}}, \forall i)$. That is, we use (10) and (27) for this comparison. We have also validated these equations through simulations. As we can see in Fig. 9, by only adding one subcarrier to transmit coded information, that is the combination of the information transmitted through the data links, we can achieve a significant performance improvement (probability of correctly receiving the information), as the number of data links increases. The performance improvement is more pronounced for high symbol error rates $(p_{\text{ser}} \geq 10^{-3})$. Moreover, DC provides excellent performance when a data link fails, e.g., d_i fails $(p_{\text{ser}_i} = 1)$.

Figs. 10 and 11 show the performance of DC-OFDM as a function of the number of data link for different, typical, code rates. As we can see in Fig. 10, 1/2, 2/3, and 3/4 code rates achieve the maximum throughput performance because the symbol error probability p_{ser} is very small. For higher symbol error probabilities, as shown in Fig. 11, DC code rate of 1/2 provides the

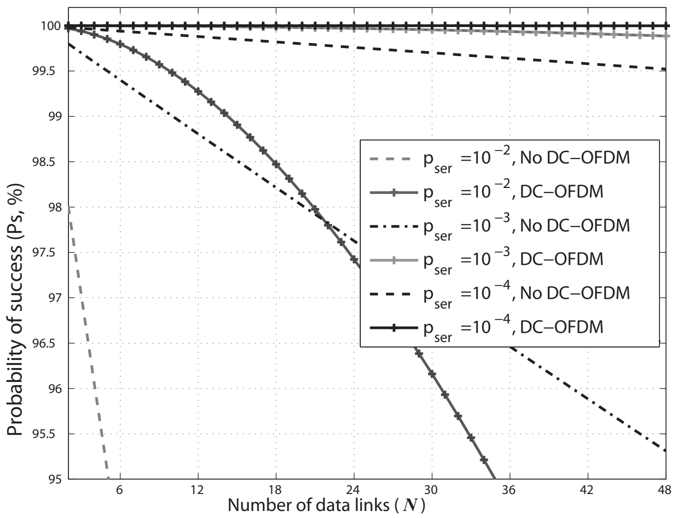


Fig. 9. Performance of a $1-for-N$ DC-OFDM system as a function of the number of subcarriers.

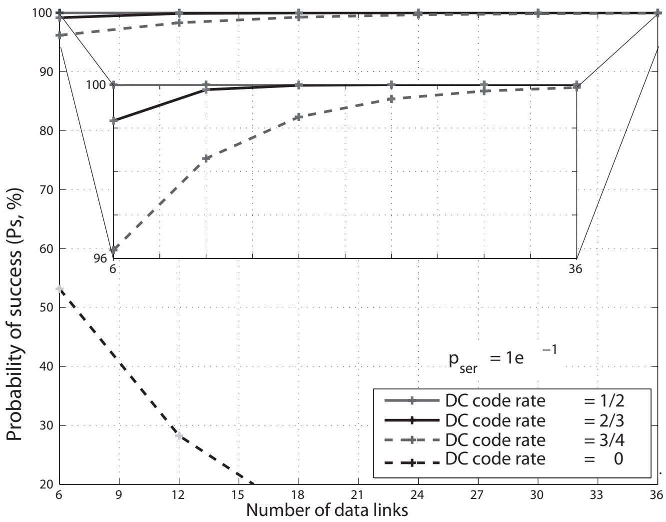


Fig. 11. Performance of a $M-for-N$ DC-OFDM system for a probability of link error $p_{\rm ser}$ of 10^{-1}.

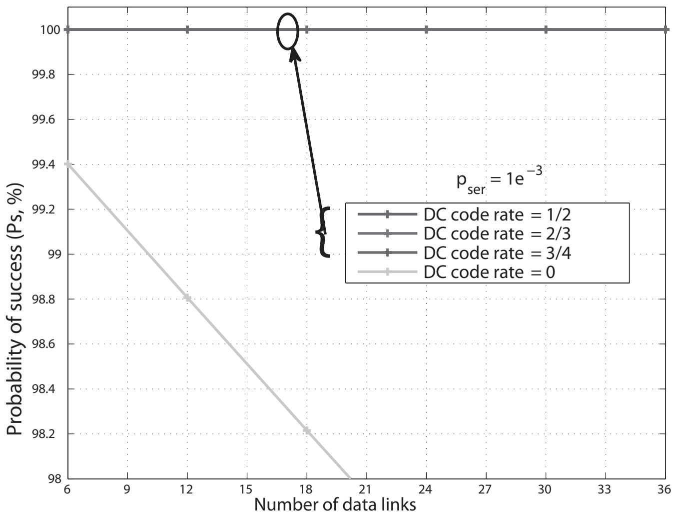


Fig. 10. Performance of a $M-for-N$ DC-OFDM system for a probability of link error $p_{\rm ser}$ of 10^{-3}.

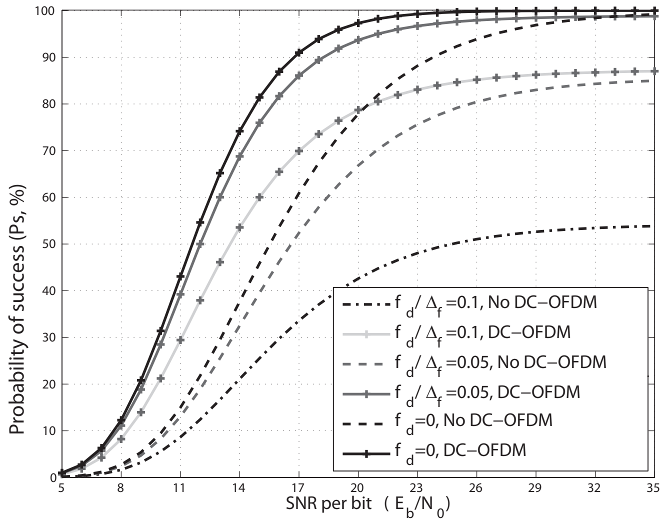


Fig. 12. Performance of $1-for-N$ DC-OFDM system for an OFDM-QPSK in a multi-path channel for various relative speeds.

highest probability of successful reception at the receiver. That is, the probability of correctly receiving the information through at least N data and/or protection lines (subcarriers). For typical symbol error probabilities ($p_{\rm ser} \approx 10^{-5}$), low DC code rates are enough to achieve the best performance. Or from another viewpoint, we can reduce the energy per symbol (or energy per bit, E_b/N_0) and increase the DC code rate to achieve a 100% of probability of successful reception.

Additionally, note that without DC as the number of data links (subcarriers) increases, the probability of correctly received information of all the data links decreases exponentially.

The performance of a $1-for-N$ DC-OFDM system for OFDM-QPSK modulated in a multi-path channel for various relative speeds between transmitter and receiver vehicles (expressed as the maximum Doppler shift (f_d) and the subcarrier spacing Δf ratio) is shown in Fig. 12. As we can see, DC-OFDM provides performance improvement for communications between stationary vehicles ($f_d = 0$). By implementing $1-for-N$ DC-OFDM, it is possible to reduce the energy per bit by about 10 dB and achieve similar performance to a system that does not use DC-OFDM. Moreover, when the relative speed between transmitter and receiver vehicles is high, the symbol error rate per subcarrier is also high. Therefore, by adding an extra subcarrier to transmit protection data, we can significantly increase the performance of the communication. Note that when the relative speed is high, it is not possible to significantly reduce the symbol error rate by increasing the energy per bit (E_b/N_0).

VI. CONCLUSION

In this paper, we proposed a DC-OFDM scheme that applies DC to OFDM-based systems such as IEEE 802.11p for vehicular environments and provides improved probability of successful reception at the receiver and transparent self-healing and fault-tolerance. DC is well suited for OFDM-based systems because of its spatial diversity nature (parallel links). DC-

OFDM provides the best performance when the probability of link error is high or when a link (sub-channel) fails. Also, by implementing DC in OFDM-based systems, we can provide reliable communication that is quite tolerant of link failures, since data and protection lines are transmitted via multiple sub-channels. Moreover, only adding one protection line (subcarrier), DC-OFDM provides significant performance improvement. Note that DC-OFDM is also well suited for mobile communications because this type of communications has high symbol error probability.

In conclusion, under typical operating conditions, DC-OFDM enables increased probability of successful reception at the receiver, thus, increasing the reliability of communications between vehicles by transmitting data and protection information through parallel subcarriers.

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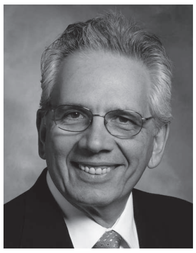

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